

# Late Hadronization and Matter Formed at RHIC: Vector Manifestation, BR Scaling and Hadronic Freedom

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## Abstract

Recent developments in our description of RHIC and related heavy-ion phenomena in terms of hidden local symmetry theories are reviewed with a focus on the novel nearly massless states in the vicinity of – both below and above – the chiral restoration temperature  $T_c$ . We present complementary and intuitive ways to understand both Harada-Yamawaki’s vector manifestation structure and Brown-Rho scaling – which are closely related – in terms of “melting” of soft glues observed in lattice calculations and join the massless modes that arise in the vector manifestation (in the chiral limit) just below  $T_c$  to tightly bound massless states above  $T_c$ . This phenomenon may be interpreted in terms of the Bég-Shei theorem. It is suggested that hidden local symmetry theories arise naturally in holographic dual QCD from string theory, and a clear understanding of what really happens near the critical point could come from a deeper understanding of the dual bulk theory. Other matters discussed are the relation between Brown-Rho scaling and Landau Fermi-liquid fixed point parameters at the equilibrium density, its implications for “low-mass dileptons” produced in heavy-ion collisions, the reconstruction of vector mesons in peripheral collisions, the pion velocity in the vicinity of the chiral transition point, kaon condensation viewed from the VM fixed point, nuclear physics with Brown-Rho scaling, and the generic feature of dropping masses at the RGE fixed points in generalized hidden local symmetry theories.

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## 1 Introduction

The discovery at RHIC of what appears to be “new matter” in the form of a strongly interacting liquid, nothing like the quark gluon plasma, brings up two issues: first, the long-standing “old” issue of what the structure of the state is in the vicinity of the presumed chiral phase transition point as well as what the proper tool to understand it is, and second, a “new” issue as to what lies above the critical point which the accepted theory of strong interactions, QCD, is supposed to be able to access perturbatively. In this review, we wish to address these issues in terms of an old idea on in-medium hadron properties proposed in 1991 [1] which has been recently rejuvenated with the surprisingly potent notion of “vector manifestation (VM)” of hidden local symmetry theory [2] and buttressed with some recent results from lattice QCD. Our principal thesis of this paper is that just as an intricate and subtle mechanism is required to reach the VM structure of chiral symmetry just below  $T_c$  – which is yet far from fully understood – from the standard linear sigma model picture applicable (and largely established) at  $T \sim 0$ , the structure of matter above and close to  $T_c$  could also be intricate and subtle from the starting point of QCD at  $T \sim \infty$  at which asymptotic freedom is applicable and “established.” We will make here a leaping extension of the ideas developed in [3] in which we infer from available information coming from lattice results and hinted by RHIC data that the structures of matter just below and just above the critical point can be related. The picture we arrive at near the chiral restoration point, as remarked in [4], resuscitates the old “Bég-Shei theorem” [5] which states: “At short distances the Nambu-Goldstone way merges with the Wigner-Weyl way: one can think of symmetry without specifying the nature of the realization.” The picture we have developed is definitely falsifiable by lattice calculations as well as by experiments. While awaiting the verdict, we shall continue exploring the implications of this highly attractive – at least to us – scenario.

As we will develop in this paper, the key to the possible new matter produced at RHIC and its connection to the matter below  $T_c$  is in the glue from the gluons exchanged between quarks and its role in chiral restoration as expressed in Brown-Rho scaling. Our first task in this article is to rephrase the Harada-Yamawaki theory “Hidden Local Symmetry at Loop” [2] in terms of some familiar results in chiral Lagrangian models in conjunction with recent lattice results which we hope will be easier to understand than the somewhat formal treatments given by Harada and Yamawaki. Our description, being more intuitive, lacks rigor but complements the gauge theoretic approach of Harada and Yamawaki. In particular, we shall interpret their results “pictorially” through the Nambu-Jona-Lasinio (NJL) theory, which is an effective theory possessing the symmetries of QCD, backed up by the results of lattice gauge calculations carried out in full QCD. We admit that there is sometimes ambiguity in how to interpret lattice gauge simulation (LGS) through the NJL model, and so we

will use empirical data to make our interpretation believable. Of course, the ultimate judge is how the Harada-Yamawaki theory describes nature<sup>1</sup>. Their theory had many initial surprises, such as “hadronic freedom” (which we shall develop more precisely later) as  $T \rightarrow T_c$  from below, whereas the seemingly equilibrated ratios of various hadrons emerging with temperature  $\sim T_c$  seemed to show that the interactions became stronger as  $T \rightarrow T_c$  from below.

One of the remarkable consequences of the notion of hadronic freedom derived from the vector manifestation fixed point of HLS is that certain processes can be more profitably described by fluctuating around the VM fixed point with vanishing mass and coupling constants instead of the standard practice of doing physics from the matter-free vacuum. We will show what this implies in processes that exhibit chiral symmetry in vacuum, the processes that take place near the chiral transition point such as kaon condensation, pion velocity, etc.

## 2 Soft Glue and the Vector Manifestation

### 2.1 Hard and Soft Glue

Originally Brown-Rho scaling [1] was proposed based on the restoration of scale invariance as  $T \rightarrow T_c$  or the density  $n \rightarrow n_c$  and was formulated in the skyrmion picture, which models QCD in the large  $N_c$  limit. This became, for a time, somewhat complicated because clearly scale invariance is still explicitly broken by the hard glue (which we often call epoxy). This is just the gluon condensate found in the initial formulation of Yang-Mills theory without quarks, the condensate which gives rise to dimensional transmutation which produces the scale  $\lambda_{QCD}$ . We now understand that the “melting” of the soft glue is responsible for the Brown-Rho scaling<sup>2</sup>.

Even before Brown-Rho scaling was proposed in 1991, Su Houn Lee [10] and Y. Deng [11] found that about half of the glue melted at  $T_c(\text{unquenched})$ , the remainder (epoxy) remaining up to beyond  $T_c(\text{quenched})$ . The separate roles

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<sup>1</sup> In [2], only the  $\rho$  meson enters as a hidden gauge field. In two recent papers, Harada and Sasaki [6] and Hidaka, Morimatsu and Ohtani [7] independently showed that the axial vector meson  $a_1$  can also be suitably incorporated into the scheme. More details will be given below.

<sup>2</sup> This point was implicit in the original formulation of Brown-Rho scaling in the 1991 paper but has remained unclear until recently. It was however clarified in various publications in different terms (e.g., identifying the soft glue with quarkonium and the hard glue with gluonium) before the lattice result [8] came along. See, e.g., the footnote 6 in ref. [9].

of the soft glue and the epoxy which remained, the isolation of the black body radiation, and other aspects were clarified in 1991 by Adami, Hatsuda, and Zahed [12] in their formulation of QCD sum rules at low temperatures.

It became clear that the soft glue, which brings about a dynamical breaking of scale invariance, was the agent building constituent quarks out of the massless (in the chiral limit) current quarks and holding them together in hadrons, whereas the hard glue, which explicitly broke scale invariance, had nothing directly to do with the hadronic masses. Thus, Brown-Rho scaling was accomplished by the restoration of the dynamically broken scale invariance by the melting of the soft glue. A theorem by Freund and Nambu [13] says that the dynamical breaking of scale invariance *requires* an explicit breaking. This shows the extreme subtlety in the interplay of explicit breaking and spontaneous breaking of scale invariance – the latter locked to quark condensates – in contrast to other global symmetries. Although no fully convincing proof exists, it is however very reasonable that the dynamical breaking is restored as  $T \rightarrow T_c(\text{unquenched})$  and the hadron masses go to zero. However, the hard glue remains far above  $T_c$ .

We now describe how to understand the renormalization group results of Harada and Yamawaki [2] from the glue calculation in LGS. Our point is simply that the glue which gives the quark its mass, making it into a constituent quark, melts as  $T \rightarrow T_c$  from below, so that the constituent quark becomes a massless current quark. Similarly, the mesonic exchange interactions between hadrons result from the exchange of soft gluons, so that hadronic interactions go to zero as  $T$  goes up to  $T_c$  from below. We will now walk the reader through these arguments using results from lattice gauge simulations to clarify our points.

Dave Miller [8] put Fig. 1 on the archives, but was unable to publish it because the referee said that “it was well known.” Yet it contains a great deal of important information, which we now discuss. (Miller is preparing a Physics Report on gluon condensates.)

We carry out our discussion within the framework of the Nambu-Jona-Lasinio (NJL) model as developed in [4]. One can think of the NJL as arising when the light-quark vector mesons (or more generally the tower of vector mesons as implied in dimensionally deconstructed QCD [14] or holographic dual QCD that arises from string theory [15,16]) and other heavy mesons are integrated out. As such, it presumably inherits all the symmetries of QCD as well as many of the results from hidden local symmetry theory, which captures the essence of QCD [2]. It is well known that in order to handle phase transitions in effective field theories, one has to treat properly the quadratic divergences that are present in loop graphs involving scalar fields. How this can be done in a chirally invariant way is explained in [2]. Now the cutoff that occurs

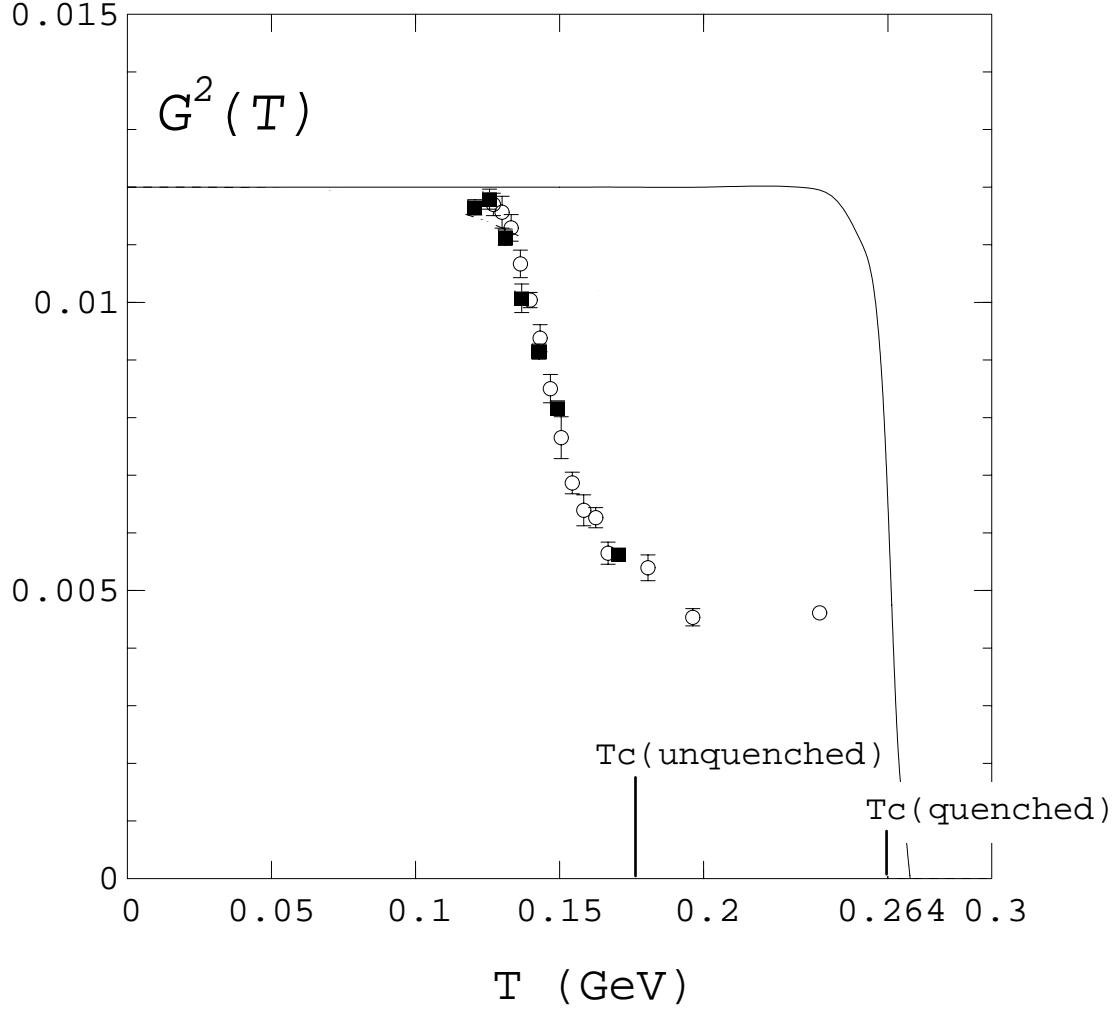


Fig. 1. Gluon condensates taken from Miller [8]. The lines show the trace anomaly for SU(3) in comparison with that of the light dynamical quarks denoted by the open circles and the heavier ones by filled circles. Note that  $G^2(T)$  ( $= -(\beta(g)/2g^3) G_a^{\mu\nu} G_{\mu\nu}^a$ ) is renormalization group invariant.

in the calculation represents the scale at which the effective theory breaks down. The natural scale for this is the chiral symmetry breaking scale  $\Lambda_{\chi SB} = 4\pi f_\pi \sim 1$  GeV. Brown and Rho [4] suggested however that the cutoff in NJL should be at  $4\pi f_\pi/\sqrt{2} \sim 700$  MeV. This cutoff was thought to be suitable for Wilsonian matching to constituent quarks rather than to current quarks. For Wilsonian matching to QCD proper, the scale has to be raised. In this regard, we are thinking in terms of a chiral quark picture where constituent quarks and (pseudo)Goldstone bosons coexist in a certain density or temperature regime. In addressing this problem, Harada and Yamawaki [2] were led to introduce hidden local gauge invariance which allowed the vector meson mass to be counted as of the same order in the chiral counting as the pion mass, without however fermion degrees of freedom. We are proposing that to be consistent with the Harada-Yamawaki theory, we need to Wilsonian-match NJL to QCD at the same scale as in HLS theory. In that case the loop graph with vector mesons comes in so as to cancel the  $\sqrt{2}$  in the NJL cutoff denominator, so the Wilsonian matching radius is raised to  $4\pi f_\pi$ .

NJL was carried out most neatly by Bernard, Meissner and Zahed [17]. We favor their results for a cutoff of  $\Lambda = 700$  MeV, which is close to  $4\pi f_\pi/\sqrt{2}$ . In BGLR [18] we got our best fits<sup>3</sup> for  $\Lambda = 660$  MeV and the NJL  $G\Lambda^2 = 4.3$  (where  $G$  is the dimensionful coupling constant). In a mean-field type of mass generation, it can be thought of as the coupling to constituent quarks of the scalar<sup>4</sup>  $\sigma$ -meson,  $G \sim -g_{\sigma QQ}^2/m_\sigma^2$ .

At  $n = 0$ ,  $T = 0$ , the proper variables are nucleons. We are sure of this from the stunning success of nuclear structure theories. They are bound states of three quarks, bound together by the glue. They have mass  $m_N$ , mostly dynamically generated from the vacuum. The degree of chiral symmetry breaking can be estimated by filling negative energy states with the nucleons down to momentum scale  $\Lambda$ . Thus

$$B(\text{glue}) = 4 \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \left\{ \sqrt{k^2 + m_N^2} - |\vec{k}| \right\} \quad (1)$$

where we have subtracted the perturbative energy  $|\vec{k}|$ . The integral is easily carried out with the result

$$B(\text{glue}) = 0.012 \text{ GeV}^4, \quad (2)$$

the value usually quoted for QCD sum rules. We used  $\Lambda = 660$  MeV.

<sup>3</sup> These parameters give  $T_c = 170$  MeV.

<sup>4</sup> Not to be confused with the  $\sigma$  that occurs in hidden local symmetry theory as the longitudinal component of the  $\rho$  meson.

As can be seen from Fig. 1, there is no melting of the glue until  $T \approx 120$  MeV. The nucleon masses are just too heavy to be pulled out of the negative energy sea by the thermal energies. But as the temperature  $T$  is increased, the nucleons will dissociate into constituent quarks. Meyer, Schwenger and Pirner [19] use a wave function which we write schematically

$$\Psi = Z|N\rangle + (1 - Z^2)^{1/2}|3q\rangle. \quad (3)$$

In other words there must be a transition of nucleons dissociating into constituent quarks as mentioned. At this stage the glue which surrounds the quarks starts to melt, and the curve for  $G^2(T)$  drops rapidly, down to  $G^2 \sim 0.0045$  at  $T_c$ . The heavy filled circles are for bare quark masses which are 4 times greater than the open MILC-collaboration ones, but the glue is insensitive to explicit chiral symmetry breaking.

We would like to suggest that the lattice results imply in dense medium a dissociation from nucleons to (colored) constituent (quasi) quarks at some density above normal nuclear matter density but below chiral restoration. This possibility was discussed a decade ago by Alkofer, Hong and Zahed [20] in terms of the NJL model where the instability of a baryon skyrmion at a density higher than normal was interpreted as splitting into  $N_c$  ( $= 3$  in nature) constituent quarks, each with an effective mass dropping as a function of density. It is not clear, though, that constituent quarks are stable propagating degrees of freedom. The idea that the constituent quark could be considered as a quark soliton (“qualiton”) was proposed a year earlier by Kaplan [21], but it turned out that no stable qualiton could be found from chiral Lagrangians so far constructed [22]. The question as to whether the constituent quarks implied in the work of [20] are bona-fide degrees of freedom near the chiral transition point as we interpret on the basis of the lattice results remains unanswered. It is interesting and intriguing to note that constituent quarks, ill-defined in QCD language, seem to find a more precise definition in holographic dual theory [23].

We can estimate the amount of soft glue that melts by changing variables from nucleons to constituent quarks, where our degeneracy factor is 12,

$$\text{B(soft glue)} = 12 \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \left( \sqrt{k^2 + m_Q^2} - |\vec{k}| \right). \quad (4)$$

Taking  $m_Q = 320$  MeV, we find  $\text{B(soft glue)} \sim \frac{1}{2}(0.012) \text{ GeV}^4$ . In the LGS the drop is a bit more than half of the  $T = 0$  glue. This could be achieved by choosing a slightly bigger constituent quark mass, say,  $m_Q \sim 370$  MeV.

So now at  $T_c$  we are left with  $G^2(T) \sim 0.006 \text{ GeV}^4$ . This is at  $T_c$  where the

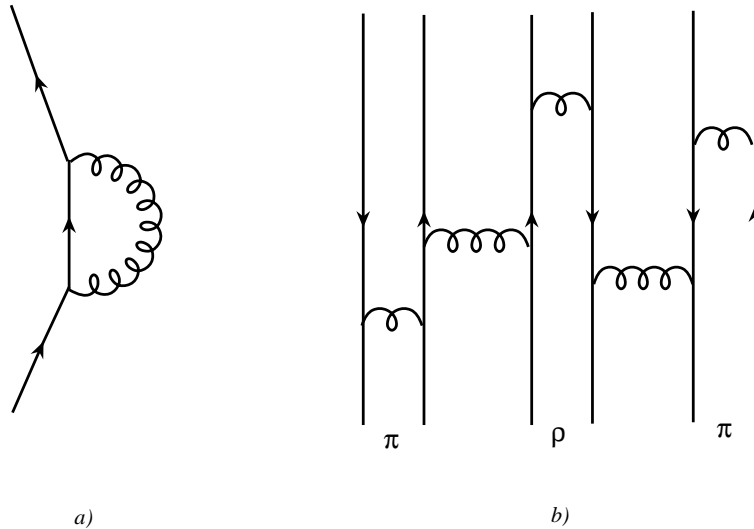


Fig. 2. Quark-gluon interactions. The wavy lines are gluons, the solid lines are quarks. In a) the gluon helps build a constituent quark out of a current quark. In b) part of the  $2\pi \rightarrow \rho$  interaction is shown.

soft glue has all melted and the constituent quarks have become (massless) current quarks. Note that the next point at  $T \sim 1.4T_c$  is equally high. There is no melting of the glue between  $T_c(\text{unquenched})$  and  $1.4T_c(\text{unquenched})$ . This is why we call this glue epoxy. It makes up the (colorless) Coulomb interaction which binds the quark-antiquark molecules above  $T_c$ . (Just above  $T_c$  it binds them to zero or nearly zero mass. See below.)

Pictorially, it is easy to see what happens. Assume that  $T > 120$  MeV, where the nucleons have dissolved into constituent quarks. These will have self energies, Fig. 2a and interactions, Fig. 2b. The constituent quark, whose mass can be thought of as the self energy, will be converted to a massless current quark as the gluon is melted. It is clear from this picture, and from the work of Harada and Yamawaki [2], that the interaction between constituent quarks, Fig. 2b, will also go to zero as the soft glue is melted. The processes in Fig. 2 result from fluctuating fields, not condensates. So they do not follow directly from the melting of the condensate, but do follow from the Harada-Yamawaki [2]  $G_V \rightarrow 0$  as  $T \rightarrow T_c$ .

In general, both the gluon and the current quark will be colored, also off-shell. Since they are only virtual particles they cannot leave the fireball until  $T = T_c$ , with increasing energy. However, they can interact with the pion, although the magnitude of the interaction will be cut down, depending on how far off shell they are. Upon reaching  $T_c$  the color could, in principle, escape. However, the



system then goes into tightly bound colorless chirally restored mesons. So, in the end, although they play a role in equilibration, energy must be furnished, in the form of a bag constant, to melt them.

The trace anomaly in the chiral limit is

$$\theta_\mu^\mu = -\frac{\beta(g)}{2g} G_a^{\mu\nu}(T) G_{\mu\nu}^a(T) \equiv G^2(T). \quad (5)$$

In fact, the contribution from quarks comes as

$$\delta\theta_\mu^\mu = \sum_q \bar{m}_q \bar{q}q, \quad (6)$$

completely from the explicit chiral symmetry breaking from the bare quark mass. The bag constant  $B$  is just

$$B = \frac{1}{4} \theta_\mu^\mu. \quad (7)$$

This shows how Brown-Rho scaling reflects the melting of the soft glue. We shall return later to a discussion of how the LGS support the scenario of meson masses going to zero as  $T \rightarrow T_c(\text{unquenched})$ . The Harada and Yamawaki work, however, goes further and shows that the width of the  $\rho$  to  $2\pi$  decay goes to zero as  $T \rightarrow T_c$  from below:

$$\Gamma(\rho \rightarrow 2\pi) \propto \left( \frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} \right)^2 \rightarrow 0 \quad (8)$$

just below  $T_c$ . With p-wave penetration factor, the power is 5 instead of 2, so it goes to zero faster. This was not foreseen by Brown and Rho in [1] but is clearly true in our scenario of the soft glue melting as  $T \rightarrow T_c$  from below, as can be seen from Fig. 2b. If the soft glue melts, then there is no transition from  $\rho$  to  $2\pi$ . In Section 4 we will describe the chirally restored mesons found above  $T_c$ , in the interval from  $T_c$  to  $\sim 2T_c$  in quenched lattice gauge calculations, and later we will try to connect these with the mesons such as  $\pi$  and  $\rho$  below  $T_c$ .

### 3 Generalized Hidden Local Symmetry (GHLS)

The vector manifestation fixed point discovered by Harada and Yamawaki [2] involved only the  $\rho$  meson as a gauge field. Now low-energy hadronic physics

relevant to matter in medium involves other “heavy” hadrons such as the vector mesons  $\omega$ ,  $a_1$  and the scalar  $\sigma$ . Indeed, in the discussion of the STAR result for the  $\rho^0/\pi^-$  ratio given below, we will invoke all these mesons in  $SU(4)$  multiplets. In confronting nature, it has been assumed up until recently that the VM fixed point behavior, such as the vanishing of the vector meson mass and the gauge coupling, applies equally to these massive mesons as one approaches the critical point. But how good is this assumption? An important part of this question has recently been answered by Harada, and Sasaki [6] and independently by Hidaka, Morimatsu and Ohtani [7], who studied what happens to the hidden local symmetry structure of effective theories near the chiral phase transition point when the  $a_1$  degrees of freedom are considered explicitly. This is an important issue in the modern development of the field in two aspects. The first is that a generalized hidden (flavor) local symmetry *naturally* arises in holographic dual QCD which emerges when the AdS/CFT conjecture is applied to nonperturbative dynamics of QCD. The second is that the  $\rho$  and  $a_1$  mesons make the (two) lowest members of the infinite tower of massive gauge fields that descend via Kaluza-Klein dimensional reduction in holographic dual QCD from a 5-D Yang-Mills Lagrangian, and their role can be understood in a more general context of QCD as a theory rather than as a particular modeling of QCD. This suggests a strong theoretical backing of the multitude of observations recently made by the authors in connection with RHIC phenomena. In this section, we present a brief discussion of these two developments.

### 3.1 Infinite tower of vector mesons

In studying physical phenomena starting from the matter free vacuum (with  $T = n = 0$ ), the local symmetry associated with the vector mesons that occur in low-energy strong interaction physics is a “luxury” [24] that one can do without. In other words, one does not have to have local symmetry when one deals with such vector mesons as  $\rho$ ,  $\omega$  and  $a_1$ . The gauge symmetry is a redundancy here. The symmetry is there since the physical field for pions  $U = e^{2i\pi/f_\pi}$  can be written in various different ways introducing local fields. For instance, Harada and Yamawaki’s HLS theory uses the definition  $U = \xi_L^\dagger \xi_R$  with  $\xi_{L,R} = e^{i\sigma(x)/f_\sigma} e^{\mp i\pi(x)/f_\pi}$ , where the  $\sigma$  field is eaten by the gauge field which becomes massive. There is a redundancy in that  $U$  is invariant under the multiplication by

$$\xi_{L,R} \rightarrow h(x) \xi_{L,R}. \quad (9)$$

This is quite analogous to what happens in other areas of physics. For instance, in condensed matter systems in which one starts with only electrons with

short-ranged interactions, there can be phases where the electron separates into a new fermion and a boson [25],

$$e(x) = b(x)f^\dagger(x). \quad (10)$$

Under the local transformations,

$$\begin{aligned} b(x) &\rightarrow e^{ih(x)}b(x), \\ f(x) &\rightarrow e^{ih(x)}f(x), \end{aligned} \quad (11)$$

the electron field remains unchanged. The new fields are redundant. One can make the symmetry a gauge symmetry by introducing gauge fields which are invisible in the original theory with the electrons. This symmetry is an emerging one. Another example which is quite analogous to this is the emergence of general coordinate invariance in the AdS/CFT duality [26].

The physics will be the same as that of a massive field with  $\sigma$  set equal to zero (corresponding to the unitary gauge) with no gauge invariance. So why all these rigmaroles with local gauge invariance and a hidden one at that? The reason is that there is a power in doing physics with the gauge symmetry kept intact that is not readily accessible in the gauge-fixed theory, and that involves going up in scale with an effective low-energy theory.

How this works out is nicely described in [27]. Imagine doing a calculation approaching the scale corresponding to that of a vector meson mass. Then having the local gauge invariance with the Goldstone boson  $\sigma$  in the Lagrangian facilitates two things. First, with the Goldstone boson, one can locate where the EFT breaks down, i.e., where “new physics” shows up: It becomes strong coupling at  $\sim 4\pi m_V/g \sim 4\pi f$  where  $f$  is the Goldstone decay constant,  $m_V$  is the vector boson mass and  $g$  is the gauge coupling. Without them, it is complicated and awkward to locate the break-down point. Now when an EFT breaks down, it is a signal that the EFT is to be “ultraviolet completed” to a fundamental theory, which in our case is QCD. In Harada-Yamawaki theory this is effectuated in some sense by the Wilsonian matching of correlators.

Second, one can systematically write higher-order terms as powers of covariant derivatives,

$$\sim \frac{1}{16\pi^2} \text{Tr}|D_\mu U|^4, \quad \sim \frac{1}{16\pi^2} \text{Tr}|D^2 U|^2, \dots \quad (12)$$

In unitary gauge, these correspond to

$$\sim \frac{1}{16\pi^2} \text{Tr} A^4, \sim \frac{1}{16\pi^2} \text{Tr} (\partial A)^2, \dots \quad (13)$$

In the absence of symmetry guidance, it is difficult to write down all the terms in the same power counting.

Although calculations may be more difficult, the above features can be accounted for without gauge invariance for phenomenology at low energy in the matter-free vacuum. This is the reason why one finds the assertion in the literature that HLS, externally gauged massive Yang-Mills and tensor field approaches are all equivalent. This assertion is correct at the tree level [2]. However, the situation is different when quantum loop effects are taken into account. In particular, suppose one wants to do “higher order” calculations, say, in chiral perturbation theory. Then if the vector meson mass needs to be considered as of the same scale as that of the pion – which is the case when Brown-Rho scaling is applicable and the vector meson mass drops low as we believe in the high  $T$  and/or high density regime – then a consistent chiral expansion involving both vector mesons and pions is feasible *only when* local gauge invariance is manifest. This point is the key point emphasized in the work of Harada and Yamawaki.

The invariance (9) involves one set of gauge fields, say,  $K = 1$  vectors  $\rho$  and  $\omega$  with  $h \in U(N_f)$ . This way of introducing gauge symmetry can be generalized to  $K > 1$  gauge fields. The generalization is not unique and the different ways of generalizing give rise to different theories. The simplest one is the “linear moose” structure on a lattice with a chain of “link fields” connecting the nearest-neighbor sites, i.e., gauge fields, labelled by  $K$  which can be extended to  $\infty$ . For instance, for  $K = 2$ , the two sites corresponding to  $\rho$  and  $a_1$  are connected to each other by one link field and to the boundaries of L and R chiral symmetries. Thus, there are towers of gauge fields for given  $K$ ’s. For a finite  $K$ , this “moose” theory can be thought of as a 4-D gauge theory on a lattice with a finite lattice spacing, and in the continuum limit with  $K \rightarrow \infty$ , it goes over to a 5-D YM theory, with the extra dimension coming from the lattice. It has been proposed that the resulting theory is a dimensionally deconstructed theory of QCD [14]. For a similar discussion in a slightly different approach, see [28].

The infinite tower of gauged vector mesons emerges also in holographic dual QCD linked to the AdS/CFT duality in string theory [15]. It has been shown that introducing quark flavors (“probe branes”) in the gravity sector in AdS space, one finds a bulk theory that is thought to correspond to QCD in the large ’t Hooft limit ( $\lambda = g\sqrt{N_C} \rightarrow \infty$ ) which comes out to be 5-D YM theory. When the fifth dimension<sup>5</sup> is compactified with suitable boundary

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<sup>5</sup> The fifth dimension is called for by holography, having to do with locality in energy in the renormalization group equation. See ref. [26] for a discussion on this

conditions, the resulting Lagrangian is found to be a hidden local symmetry theory (denoted in short as HDHLS for “holographic dual hidden local symmetry”) with an infinite tower of vector mesons. This is an effective theory valid below a Kaluza-Klein cutoff  $M_{KK}$ , the compactification scale, possessing spontaneously broken chiral symmetry and chiral anomalies (Wess-Zumino-Witten term) of QCD. In going from 5-D to 4-D, the 5th component of the gauge field is arbitrary. It turns out to be convenient to gauge fix it to zero to make contact with Harada-Yamawaki HLS. However, if it is gauge-fixed to the pion field, *all* low-energy hadron processes, strong as well as responses to the electroweak field, are found to be manifestly vector dominated. Thus vector dominance (VD) is universal and automatic in this theory. Surprisingly a variety of low-energy relations such as KSRF, GMOR, etc. also come out correctly in this theory.

Another important aspect of the theory is that HDHLS, comprising entirely of vector mesons and Goldstone bosons, has no fermion degrees of freedom. This means that the ground-state baryons and their excited states must arise as topological solitons, i.e., skyrmions, in 4-D theory or instantons in 5-D theory. It is intriguing that the skyrmion is an indispensable ingredient in this HDHLS theory. Therefore, in order to study dense matter where baryon density must be taken into account, skyrmions have to be considered. This point has been stressed in [29] but in terms of the Skyrme Lagrangian, which we now know is not realistic without incorporating vector mesons. It is likely that hidden local gauge fields can provide topological order not present in the non-gauged skyrmions [30].

An interesting open problem is whether the  $K = 1$  theory of Harada and Yamawaki, or the  $K = 2$  GHLS theory with  $a_1$ , can be understood in terms of a truncated theory of a holographic dual HLS theory. The latter predicts that the electromagnetic form factors of the pion as well as the nucleon will be vector-dominated involving the infinite tower of the vector mesons. In nature, it is known empirically that the pion form factor is vector dominated by the  $K = 1$  vectors but that the nucleon form factor is not dominated by the lowest member of vector mesons. It is not surprising that the vector dominance involving an infinite tower of vector mesons could be violated when the space is truncated to the lowest members, with the violation representing the effect of the integrated-out vector mesons. It is, however, intriguing that vector dominance with the  $K = 1$  vector mesons holds so well for the pion form factors while it does not for the nucleon form factors. It would be interesting to investigate whether this rather special empirical observation follows from HDHLS by integrating out the higher-lying members of the tower. Such a study is in progress.

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point.

### 3.2 How does the $a_1$ figure?

For explaining some of the RHIC observations, e.g., the STAR  $\rho^0/\pi^-$  ratio discussed below, in addition to the pions and the  $K = 1$  vector mesons ( $\rho$  and  $\omega$ ) other more massive mesons in flavor  $SU(4)$  symmetry need to be accounted for near the critical region. As a first step to see how other degrees of freedom enter in the fixed point structure of the EFT under consideration, the role of  $a_1$  has recently been elucidated independently by two groups, Harada and Sasaki [6] and Hidaka, Morimatsu and Ohtani [7]. It is found in this “generalized hidden local symmetry” theory that as the order parameter of chiral symmetry, i.e., the quark condensate  $\langle \bar{q}q \rangle$ , goes to zero (in the chiral limit), there can be three different fixed points with  $g = 0$  characterized by

$$\begin{aligned} \text{GL-type} &: M_\rho^2/M_{a_1}^2 \rightarrow 1, \\ \text{VM-type} &: M_\rho^2/M_{a_1}^2 \rightarrow 0, \\ \text{Hybrid-type} &: M_\rho^2/M_{a_1}^2 \rightarrow 1/3. \end{aligned} \quad (14)$$

Here the “GL (Ginzburg-Landau)-type” corresponds to the standard sigma model scenario, the “VM-type” corresponds to the Harada-Yamawaki vector manifestation scenario, and the “hybrid-type” is a new scenario that will be clarified below. These types are characterized by different multiplet structures. To specify them, write the representations of the scalar, pseudoscalar, longitudinal vector and axial vector mesons as

$$\begin{aligned} |s\rangle &= |(N_f, N_f^*) \oplus (N_f^*, N_f)\rangle, \\ |\pi\rangle &= |(N_f, N_f^*) \oplus (N_f^*, N_f)\rangle \sin \psi \\ &\quad + |(1, N_f^2 - 1) \oplus (N_f^2 - 1, 1)\rangle \cos \psi, \\ |\rho\rangle &= |(1, N_f^2 - 1) \oplus (N_f^2 - 1, 1)\rangle, \\ |a_1\rangle &= |(N_f, N_f^*) \oplus (N_f^*, N_f)\rangle \cos \psi \\ &\quad - |(1, N_f^2 - 1) \oplus (N_f^2 - 1, 1)\rangle \sin \psi, \end{aligned} \quad (15)$$

where  $\psi$  denotes the mixing angle. Including the representation, the fixed points are characterized by

$$\begin{aligned} \text{GL-type} &: \cos \psi \rightarrow 0, \\ \text{VM-type} &: \sin \psi \rightarrow 0, \\ \text{Hybrid-type} &: \\ &\sin \psi \rightarrow \sqrt{\frac{1}{3}}, \quad \cos \psi \rightarrow \sqrt{\frac{2}{3}}. \end{aligned} \quad (16)$$

It comes out that these different fixed points predict different couplings to the electromagnetic field. This is highly relevant for phenomenological tests, e.g., in dilepton productions discussed below. For instance, near the chiral phase transition point the  $\gamma\pi\pi$  coupling is predicted to approach

$$\begin{aligned} \text{GL-type} &: g_{\gamma\pi\pi} \rightarrow 0, \\ \text{VM-type} &: g_{\gamma\pi\pi} \rightarrow \frac{1}{2}, \\ \text{Hybrid-type} &: g_{\gamma\pi\pi} \rightarrow \frac{1}{3}. \end{aligned} \tag{17}$$

Note that in the GL-type, VD gets restored as chiral symmetry is restored, in stark contrast to the VM-type case for which VD is strongly violated. The hybrid type also gives about 33% violation of the VD since the direct  $\gamma\pi\pi$  comes to be 1/3.

Now there are three fixed points, all consistent with chiral restoration given by the RGE flow of the generalized hidden local symmetry. The natural question is which fixed point is chosen by nature when the system is driven to chiral restoration. In order to answer this question, one would have to study the theory at high temperature, density and with an increasing number of flavors. Work is in progress on this matter. But a priori, there is nothing to indicate that only one of them will be reached in nature by all three conditions, high temperature, high density and high number of flavors. Indeed it appears that the states above  $T_c$ ,  $n_c$  and  $N_f^c$  are of basically different nature, and furthermore, if other soft modes such as kaon condensation appear before the chiral restoration, the transition to the “chirally restored” state will be drastically modified from what is described by an effective theory of HLS-type [30].

#### 4 Chirally Restored Mesons, Equivalently $\bar{q}q$ Bound States, From $T_c$ To $2T_c$

The question we wish to address next is why and how the mesons with chiral symmetry spontaneously broken below  $T_c$  reappear above  $T_c$ . We shall see below that the  $\pi, \sigma, \rho$ , and  $a_1$ , which form a badly broken SU(4) below  $T_c$ ,<sup>6</sup> are

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<sup>6</sup> Here  $SU(4)$  is “spin $\otimes$ isospin” symmetry. In this spin-isospin-symmetry-restored phase, we use the extended notation  $\pi \equiv (\pi, \eta)$  (pseudoscalar particles),  $\sigma \equiv (\sigma, \delta = a_0)$  (scalar particles),  $\rho \equiv (\rho, \omega)$  (vector particles),  $a_1 \equiv (a_1, \epsilon = f_1)$  (axial-vector particles). Therefore, the number of meson states is  $2(\text{chiral symmetry}) \times 4(\text{spin}) \times 4(\text{isospin}) = 32$  with massive vector particles. For the massless case, we have only two polarization states of vector particles, so the number of states is  $32 \times 3/4 = 24$ . In our estimates, however, due to the fine-splitting which is caused by the spin-spin

nearly degenerate in the region between  $T_c(\text{unquenched})$  and  $T_c(\text{quenched})$ , a region of  $175 \text{ MeV} < T < 250 \text{ MeV}$ . Brown et al. [31] argue that mesons, rather than liberated quarks and gluons, are the correct variables up to beyond  $T_c(\text{quenched})$  and that above  $T_c(\text{unquenched}) = T_{\chi SB}$  the (hard) glue remains condensed. Thus, although mesons can be regarded as quark-antiquark pairs, each quark must be connected with the antiquark in the meson by a “string” (i.e., a line integral of the vector potential; equivalently, a Wilson line) in order to preserve gauge invariance. It was then noted that it is difficult to include consequences of the line integral in the thermodynamic development of the system. Indeed, the Bielefeld LGS [32] have found that for temperatures  $\gtrsim T_c$ , the confining properties of the heavy quark potential are just those of the  $T = 0$  charmonium potential including string tension. Now, in fact our very small mesons, with rms radius  $\sim 0.2 \text{ fm}$ , are bound just above  $T_c$  by the Coulomb plus magnetic interactions, but were they not, the string tension would act as a backup to keep them small, inside of the  $\sim 0.5 \text{ fm}$  screening radius. In the above sense, all that happens with chiral restoration is that the masses of the mesons go to zero.

We shall consider throughout only the mesons calculated in Nambu-Jona-Lasinio, i.e.,  $\pi, \sigma, \rho$  and  $a_1$  of the  $SU(4)$  multiplet. These are collective excitations, analogous to nuclear vibrations, in that their wave functions have many coherent components; i.e., the wave functions involve sums over particle-hole excitations, with momenta limited by a cutoff  $\Lambda$ , where  $\Lambda$  can be viewed as the Wilsonian matching scale for constituent quarks. This is why these mesons tend to have strong couplings. We believe that they are responsible for nearly all of the thermodynamics, although there are many interesting effects such as strangeness equilibration, etc., with which we do not deal.

The RHIC material should be ideally suited for descriptions in LGS. There are nearly as many antiparticles as particles, so the baryon number is small. (It is zero in LGS.) Equilibration has been shown to be good at RHIC, at least down to  $T_c$ . The lattice calculations of the Coulomb potential are for heavy quarks, actually quarks of infinite mass. We are interested in light-quark systems,  $\pi, \sigma, \rho, a_1$ , for which we have to add magnetic effects. As pointed out in BLRS [33], this can be done by going back to the work of Brown [34], who showed that for stationary states of two  $K$ -electrons in heavy atoms the interaction was of the form

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interaction, the vector particles maintain a finite but small mass. Now, the question is what happens when we approach the chiral restoration temperature from below, where the vector-meson mass vanishes. We do not have a clear answer for this question. In our approach, we assume that the vector meson mass approaches zero but remains finite until the chiral phase transition, at which point our mesonic bound states take over.



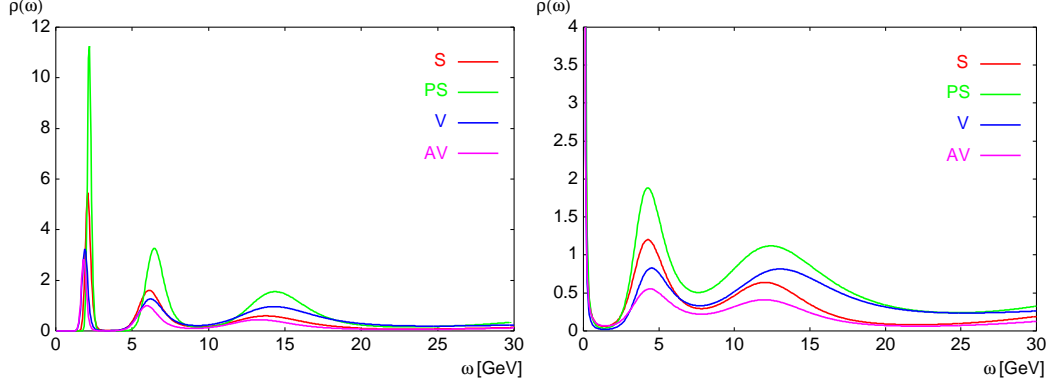


Fig. 3. Spectral functions of Asakawa et al. [37]. Left panel: for  $N_\tau = 54$  ( $T \simeq 1.4T_c$ ). Right panel: for  $N_\tau = 40$  ( $T \simeq 1.9T_c$ ).

$$V_c = \frac{\alpha}{r} (1 - \vec{\alpha}_1 \cdot \vec{\alpha}_2), \quad (18)$$

where the  $\vec{\alpha}$ 's are the Dirac velocity operators. Since helicity is good above  $T_c$ , as nicely explained by Weldon [35],  $\vec{\alpha}_1 \cdot \vec{\alpha}_2 = \pm 1$ . Now applying this to QCD, with opposite helicities for quark and antiquark, we will have

$$V_c = \frac{2\alpha_s}{r} \quad (19)$$

and for the same helicity  $V_c = 0$ . The latter can then be neglected for  $T \sim T_c$ .

Shuryak and Zahed [36] considered the region of  $T \sim 2T_c$ , where the  $\bar{q}q$  bound states break up. We show in Fig. 3 the lattice gauge calculations of the spectral functions of Asakawa et al. [37] for  $T = 1.4T_c$  and for  $1.9T_c$ . We consider only the lowest excitations in each case. Note that all SU(4) excitations are essentially degenerate. However, the  $T = 1.9T_c$  excitations at  $\omega \sim 4.5$  GeV are rather broad and it is clearly around here, roughly at the temperature formed initially at RHIC, that the  $\bar{q}q$  pairs are breaking up. Now as they go through zero binding the  $\bar{q}q$  scattering amplitude goes from  $\infty$  to  $-\infty$  and the interaction becomes very strong, as suggested by Shuryak and Zahed [36]. In their formation the quark and antiquark velocities go to zero at breakup, implying that our result Eq. (18) holds only for  $T_c$  where the quark velocity is 1, the  $\pi$  and  $\sigma$  being massless. The result is that the viscosity is very low and this is the material which is called “perfect liquid” [38]. We do not have more to say other than that this is the upper end, in temperature, of our  $\pi, \sigma, \rho, a_1$  set of mesons. They come unbound into quarks and antiquarks here. Of course the colorless states we work with are mixed into a multitude of other states, colorless and colored, by  $T = 1.9T_c$ . This situation is quite analogous to the neutron giant resonances in low-energy nuclear physics [39].

One might conclude from the quenched lattice results shown in Fig. 3 that all 32 excitations labeled, S, PS, V and A are degenerate. In fact, this is not

entirely true. Chiral symmetry restoration above  $T_c$  does guarantee that the  $\pi$  (in the chiral limit) and  $\sigma$  mesons are degenerate. However, the  $\rho$ -meson will be slightly higher in energy than the  $\pi$  and  $\sigma$ , as will be the  $a_1$ , which is equivalent to the  $\rho$  above  $T_c$ . The  $\rho$  and  $a_1$  have spin 1, the spin interaction lifting their degeneracy with the  $\pi$  and  $\sigma$ . However, their magnetic moments

$$\mu_{q,\bar{q}} = \mp \frac{\sqrt{\alpha_s}}{p_0} \quad (20)$$

are strongly suppressed by the large magnitude of the thermal mass which enters into  $p_0$

$$p_0 = E + m_{\text{th}} \quad (21)$$

in the region of large thermal masses just above  $T_c$ . In fact, we estimate that the  $\rho$  and  $a_1$  will be shifted above the  $\pi$  and  $\sigma$  by

$$\Delta E = m_{\text{th}}/6 \quad (22)$$

just above  $T_c$ .

## 5 Lattice Gauge Calculations in Full QCD

LGS have been carried out in full QCD for  $SU(2) \times SU(2)$  and are published in two parts by O. Kaczmarek and F. Zantow [40,41]. The first part [40] concerns a discussion of the quark-antiquark free energies and zero temperature potential in two-flavor QCD. The second part [41] concerns a detailed discussion of the lattice data for the color singlet quark-antiquark internal energies.

Aside from rescaling the temperature from the earlier Bielefeld quenched calculations to unquenched, the results are not very different from the earlier calculations. This is perhaps not surprising since the color singlet (Coulomb) interaction dominates the phenomena; i.e., the color singlet gluon mode runs the show.

Park, Lee and Brown [42] showed that at  $T_c$  putting the earlier quenched Bielefeld LGS results into a Klein-Gordon equation and doubling it in order to take into account the magnetic (Ampere's Law) interaction as shown in the  $\vec{\alpha}_1 \cdot \vec{\alpha}_2$  term in Eq. (18), the masses of the 32 degrees of freedom shown in their Fig. 4 went to zero as  $T$  went down to  $T_c$ . They did not, however, include the spin-dependent interaction that involves the  $\mu_{q,\bar{q}}$  of Eq. (20). It is unclear what takes place exactly at  $T_c$ ; indeed Harada and Yamawaki warned against

sitting on the fixed point [2]. However, the thermal mass  $m_{\text{th}}$  is very large ( $> 1$  GeV) when calculated in the quenched approximation by Petreczky et al. [43] at  $T = 1.5T_c$ , not far above  $T_c \sim 170$  MeV, and one expects the quark “mass” (more precisely the fourth component of the four momentum) to go to  $\infty$  with confinement. Thus a reasonable assumption is that as  $T$  goes down to  $T_c$  from above, the spin-dependent effects go to zero at  $T_c$ , because they are inversely proportional to the “mass” (containing the large thermal mass), and the apparent SU(4) symmetry seen in the LGS above  $T_c$  goes, just at  $T_c$ , over into an exact SU(4). We give in the next section arguments from experiment that this is true to the accuracy with which experiment can measure it, and we show that this can explain what has been a surprising result up to now.

## 6 The STAR $\rho^0/\pi^-$ ratio

As an illustration of the potency of the notion of “hadronic freedom,” we sketch the surprisingly simple argument that the STAR  $\rho^0/\pi^-$  ratio in peripheral  $Au + Au$  collisions at RHIC is explained if the  $\pi, \sigma, \rho, a_1$  chirally restored mesons seen in LGS above  $T_c$  persist down through  $T_c$ , where they are essentially massless, remain dormant until the temperature is low enough for them to go back  $\sim 90\%$  on shell, and then decay into pions. This is the extension of the  $\rho \rightarrow 2\pi$  decay described by Shuryak and Brown [44], in which it was shown that the  $\rho$ -meson decayed into two pions at  $T \sim 120$  MeV, thermal freezeout for the peripheral experiments. The  $\rho^0$  could be reconstructed from the two pions, and it was shown that the  $\rho$  mass had decreased  $\sim 10\%$ , some of the decrease from Boltzmann factors, but about 38 MeV from Brown-Rho scaling; i.e., as a medium effect coming from the scalar densities furnished by the baryons and by the vector mesons. The  $\rho$ -meson at the freezeout temperature was only 10% off-shell.

The result of STAR, after reconstructing the  $\rho$ -mesons by following pion pairs back to their origin in the time projection chamber, was that at  $T \sim 120$  MeV the  $\rho^0/\pi^-$  ratio was

$$\left. \frac{\rho^0}{\pi^-} \right|_{\text{STAR}} = 0.169 \pm 0.003(\text{stat}) \pm 0.037(\text{syst}), \quad (23)$$

almost as large as the  $\rho^0/\pi^- = 0.183 \pm 0.001(\text{stat}) \pm 0.027(\text{syst})$  in proton-proton scattering. The near equality of these ratios was not expected<sup>7</sup>, since the  $\rho$  meson width of  $\Gamma \sim 150$  MeV in free space is the strongest meson

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<sup>7</sup> One might argue that the near equality means just that nothing unusual with respect to pp scattering happens in the STAR process, the ratio in heavy-ion collisions being about the same as in pp scattering. Our description however involves

Table 1

$\Gamma^*$  as function of temperature. For the point at 120 MeV we have switched over to the Shuryak and Brown [44] value for  $\Gamma^*/\Gamma|_{\rho \rightarrow 2\pi}$ .

T	$m_\rho^*/m_\rho$	$\Gamma^*/\Gamma$
175 MeV	0	0
164 MeV	0.18	0
153 MeV	0.36	0.01
142 MeV	0.54	0.05
131 MeV	0.72	0.22
120 MeV	0.90	0.67

rescattering that there is. If one assumes equilibrium at freezeout, then the ratio is expected to be [45]

$$\rho^0/\pi^- \sim 4 \times 10^{-4}. \quad (24)$$

We now show that the (nearly) massless  $\pi, \sigma, \rho, a_1$  at  $T_c$  remain dormant until the temperature drops to  $T \sim 120$  MeV, which is the *freezeout temperature* for the peripheral collisions. There the temperature, which we call the *flash temperature*  $T_{\text{flash}}$ , is such that the mesons go sufficiently close to being on shell and their vector coupling approaches sufficiently close to the free space strength that they can decay into pions. Given our  $\pi, \sigma, \rho, a_1$  the  $\rho^0/\pi^-$  ratio comes out close to the empirical value. This shows that just below  $T_c$  one has what we call “hadronic freedom,” the  $\pi, \sigma, \rho, a_1$  remaining dormant. Then, at the flash temperature  $T_{\text{flash}}$  the  $\sigma, \rho$  and  $a_1$  decay into pions, and it is a simple question of counting in order to obtain the STAR  $\rho^0/\pi^-$  ratio.

We first work around  $T_{\text{flash}}$  in order to show how nicely our considerations here fit in with the Shuryak and Brown analysis [44]. These authors noted that in the movement downwards to  $T_{\text{freezeout}} \approx 120$  MeV the width of the  $\rho$ -meson did not seem to change, although there should be a kinetic effect. The negative mass shift automatically reduces (kinematically) the width, both because of the reduced phase space and also due to the power of  $p$  in the  $p$ -wave matrix element. This kinematic shift in width should go as inverse third power in shift in mass.

Now with the Harada-Yamawaki VM effect – i.e., the intrinsic background

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the very subtle notion of “hadronic freedom” based on the vector manifestation. This near equality could very well be coincidental and could not be taken as an evidence against our scenario, particularly since the ratio in pp scattering has not yet been explained by particle theorists.

dependence – taken into account, the width must drop even faster as<sup>8</sup>

$$\frac{\Gamma_\rho^*}{\Gamma_\rho}|_{\rho \rightarrow 2\pi} \sim \left(\frac{m_\rho^*}{m_\rho}\right)^3 \left(\frac{g_V^*}{g_V}\right)^2 \Rightarrow \left(\frac{m_\rho^*}{m_\rho}\right)^5, \quad (25)$$

the dropping in  $(g_V^*/g_V)$ , from loop correction, beginning only a bit higher up than  $T_{\text{flash}} = 120$  MeV. If we use the scaling with third power for  $T = 120$  MeV and that with fifth power above, we get the results in Table 1 for  $\Gamma^*$  as function of temperature. We have let the mass drop linearly with temperature as indicated by the more or less linear drop in Fig. 1 of the soft glue with temperature. From Table 1 we see that at  $T = T_{\text{flash}} = 120$  MeV the width  $\Gamma^*$  goes 2/3 of the way back to the on-shell 150 MeV. The movement back towards the on-shell value is rather sudden. This is why we call 120 MeV the flash temperature. Thus, in the neighborhood of  $T = 120$  MeV the mesons other than the pions, which are already present, decay into pions.

Now the number of pions coming off at  $T_{\text{flash}}$  is just a question of counting. Once the meson is nearly on-shell it decays rapidly. We find that in total 66 pions result at the end of the first generation from the 32  $SU(4)$  multiplet, i.e.,  $\rho$  (18),  $a_1$  (27),  $a_0$  (4),  $\pi$  (3),  $\sigma$  (2) and  $\epsilon \equiv f(1285)$  (12), where the number in the parenthesis is the number of pions emitted. Excluded from the counting are the  $\omega$  and  $\eta$  since they leave the system before decaying. Leaving out the three  $\pi^-$ 's coming from the  $\rho^0$  decays which are reconstructed in the measurement, we obtain

$$\frac{\rho^0}{\pi^-} \approx 3/(22 - 3) \approx 0.16. \quad (26)$$

We can understand this large ratio (very large compared with the equilibrium  $4 \times 10^{-4}$ ) by the fact that the mesons go through a hadron free region until they decay, never equilibrating. With the same  $\pi, \sigma, \rho, a_1$  that we see above  $T_c$  in the quenched LGS, we find the right number of pions are emitted to fit the STAR experiment.

We should stress that whereas  $T_{\text{flash}}$  is independent of centrality,  $T_{\text{freeze out}}$  is lower for higher centrality. Thus,  $\rho$ 's cannot be reconstructed from the central collisions in STAR. The reason is simple: the pions which they have decayed into will have suffered rescatterings following the decay.

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<sup>8</sup> The collisional width which we are not considering here should also drop since the  $a_1\rho\pi$  coupling constant is also proportional to the gauge coupling  $g_V$  which drops.

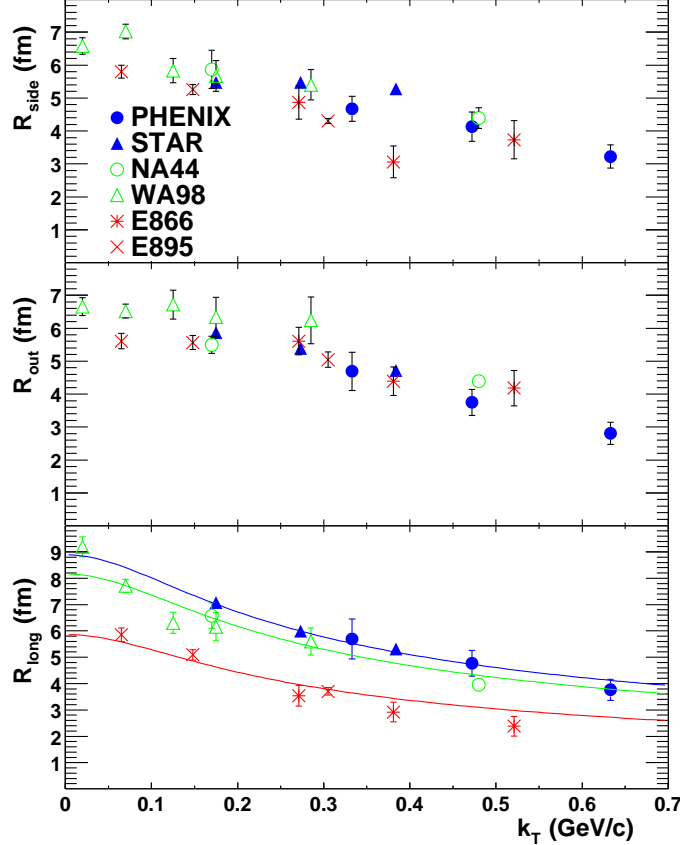


Fig. 4. The measurements of HBT radii for pion pairs (taken from Fig. 2 of Adcox et al. [46]) by PHENIX [46], STAR [47], NA44 [48], WA98 [49], E866 [50] and E895 [51]. These show all three radii to be essentially the same. The bottom plot includes fits to the data. The data are for  $\pi^-$  results except for the NA44 results, which are for  $\pi^+$ .

## 7 Comments on Hanbury Brown-Twiss Puzzle

We have given a detailed description of the dynamics of what is usually called the “mixed phase,” at least of the phase from  $T_c$  down to  $T_{\text{flash}}$ . For the peripheral collisions involved here, the pions are emitted by the vector and axial vector mesons and then leave the system at  $T_{\text{freezeout}}$  without interacting with the fireball as a whole. It is hard to see how these will carry information about the latter.

With respect to central collisions the situation is different, in that the pions are emitted for temperatures greater than  $T_{\text{freezeout}}$ . We suggest that there may still be some influence on the HBT puzzle because of how the dynamics are affected by the vector and axial-vector mesons having to go on-shell before they can interact (and decay). This means that in the entire mixed phase from  $T = T_c \simeq 175$  MeV down to  $T \simeq 120$  MeV the expansion will be at nearly the velocity of light  $c$ , which it begins with at  $T_c$ . This is because due to hadronic

freedom there is essentially no interaction with the off-shell mesons until  $T$  has dropped to  $\sim 120$  MeV, only with a few of the melted soft gluons. This means that the system can increase in radius  $\sim 5$  fm, one fermi for each interval in Table 1. Thus, the system will not be very far from spherical, at least pumpkin shaped, by the time the pions begin interacting.

Once the vector and axial-vector mesons are on-shell at  $T \sim 120$  MeV, there will be an explosion because their interactions are suddenly turned on. Given an exploding, nearly spherical system of large radius, most natural would seem to be outward, sideways and longitudinal radii which are similar (See Fig. 4). In any case we believe that the HBT calculations should be carried out starting from our detailed dynamics.

## 8 The Pion Velocity at $T_c$

There are several quantities measured in laboratory experiments and lattice simulations that can eventually be checked against. Here we treat the pion velocity. Possible deviation from the velocity of light for the massless pion (in the chiral limit) is expected since Lorentz invariance is broken in a heat bath. The relevant Lagrangian with Lorentz symmetry broken is

$$\begin{aligned} \tilde{L} = & \left[ (F_{\pi,\text{bare}}^t)^2 u_\mu u_\nu + F_{\pi,\text{bare}}^t F_{\pi,\text{bare}}^s (g_{\mu\nu} - u_\mu u_\nu) \right] \text{tr} [\hat{\alpha}_\perp^\mu \hat{\alpha}_\perp^\nu] \\ & + \left[ (F_{\sigma,\text{bare}}^t)^2 u_\mu u_\nu + F_{\sigma,\text{bare}}^t F_{\sigma,\text{bare}}^s (g_{\mu\nu} - u_\mu u_\nu) \right] \text{tr} [\hat{\alpha}_\parallel^\mu \hat{\alpha}_\parallel^\nu] \\ & + \left[ -\frac{1}{g_{L,\text{bare}}^2} u_\mu u_\alpha g_{\nu\beta} - \frac{1}{2g_{T,\text{bare}}^2} (g_{\mu\alpha} g_{\nu\beta} - 2u_\mu u_\alpha g_{\nu\beta}) \right] \text{tr} [V^{\mu\nu} V^{\alpha\beta}] + \dots, \end{aligned} \quad (27)$$

where  $F_{\pi,\text{bare}}^t$  ( $F_{\sigma,\text{bare}}^t$ ) and  $F_{\pi,\text{bare}}^s$  ( $F_{\sigma,\text{bare}}^s$ ) denote the *bare* parameters associated with the temporal and spatial decay constants of the pion (of the  $\sigma$ ). Here  $u = (1, 0)$  is the unit four-vector for the rest frame. The parameters of the Lagrangian are the “bare” ones determined at the matching point by matching HLS correlators to QCD ones. We recall here that, due to the Lorentz symmetry violation, the two variables  $\xi_L$  and  $\xi_R$  included in the 1-forms  $\hat{\alpha}_\perp^\mu$  and  $\hat{\alpha}_\parallel^\mu$  in Eq. (27) are parameterized as

$$\xi_{L,R} = e^{i\sigma/F_\sigma^t} e^{\mp i\pi/F_\pi^t}, \quad (28)$$

where  $F_{\pi,\text{bare}}^t$  and  $F_{\sigma,\text{bare}}^t$  are the bare parameters associated with the temporal decay constants of the pion and the  $\sigma$ .

We also need the terms of  $O(p^4)$  for the present analysis:

$$\bar{\mathcal{L}}_{z_2} = \left[ 2z_{2,\text{bare}}^L u_\mu u_\nu g_{\nu\beta} + z_{2,\text{bare}}^T (g_{\mu\alpha} g_{\nu\beta} - 2u_\mu u_\alpha g_{\nu\beta}) \right] \text{tr} \left[ \hat{\mathcal{A}}^{\mu\nu} \hat{\mathcal{A}}^{\alpha\beta} \right] , \quad (29)$$

where the parameters  $z_{2,\text{bare}}^L$  and  $z_{2,\text{bare}}^T$  correspond in medium to the vacuum parameter  $z_{2,\text{bare}}$  [2] at  $T = \mu = 0$ .  $\hat{\mathcal{A}}^{\mu\nu}$  is defined by

$$\hat{\mathcal{A}}^{\mu\nu} = \frac{1}{2} \left[ \xi_R \mathcal{R}^{\mu\nu} \xi_R^\dagger - \xi_L \mathcal{L}^{\mu\nu} \xi_L^\dagger \right] , \quad (30)$$

where  $\mathcal{R}^{\mu\nu}$  and  $\mathcal{L}^{\mu\nu}$  are the field-strength tensors of the external gauge fields  $\mathcal{R}_\mu$  and  $\mathcal{L}_\mu$ :

$$\begin{aligned} \mathcal{R}^{\mu\nu} &= \partial^\mu \mathcal{R}^\nu - \partial^\nu \mathcal{R}^\mu - i [\mathcal{R}^\mu, \mathcal{R}^\nu] , \\ \mathcal{L}^{\mu\nu} &= \partial^\mu \mathcal{L}^\nu - \partial^\nu \mathcal{L}^\mu - i [\mathcal{L}^\mu, \mathcal{L}^\nu] . \end{aligned} \quad (31)$$

Now define the parametric  $\pi$  and  $\sigma$  velocities as

$$V_\pi^2 = F_\pi^s / F_\pi^t, \quad V_\sigma^2 = F_\sigma^s / F_\sigma^t. \quad (32)$$

The approach to the chiral restoration point should be characterized by the equality between the axial-vector and vector current correlators in QCD,  $G_A - G_V \rightarrow 0$  for  $T \rightarrow T_c$ . The EFT should satisfy this also for any values of  $p_0$  and  $\bar{p}$  near the matching point provided the following conditions are met:  $(g_L, g_T, a_{\text{bare}}^t, a_{\text{bare}}^s) \rightarrow (0, 0, 1, 1)$  for  $T \rightarrow T_c$ . As in dense medium, this implies that at the tree or bare level, the longitudinal mode of the vector meson becomes the real NG boson and couples to the vector current correlator, while the transverse mode decouples. A non-renormalization theorem by Sasaki [52] shows that  $(g_L, a^t, a^s) = (0, 1, 1)$  is a fixed point of the RGEs satisfied at any energy scale. Thus the VM condition is given by

$$(g_L, a^t, a^s) \rightarrow (0, 1, 1) \quad \text{for} \quad T \rightarrow T_c. \quad (33)$$

The VM condition for  $a^t$  and  $a^s$  leads to the equality between the  $\pi$  and  $\sigma$  (i.e., longitudinal vector meson) velocities:

$$(V_\pi / V_\sigma)^4 = (F_\pi^s F_\sigma^t / F_\sigma^s F_\pi^t)^2 = a^t / a^s \xrightarrow{T \rightarrow T_c} 1. \quad (34)$$

This is easy to understand in the VM scenario since the longitudinal vector meson becomes the chiral partner of the pion. This equality holds at  $T_c$  whatever the value of the bare pion velocity obtained at the matching point.



- *The standard sigma model scenario*

Before we get into the discussion on the HLS prediction, it is instructive to see what we can expect in chiral models without light vector meson degrees of freedom. Here the basic assumption is that near chiral restoration, there is no instability in the channel of the degrees of freedom that have been integrated out. In this pion-only case, the appropriate effective Lagrangian for the axial correlators is the in-medium chiral Lagrangian dominated by the current algebra terms,

$$L_{eff} = \frac{f_\pi^2}{4} \left( \text{Tr} \nabla_0 U \nabla_0 U^\dagger - v_\pi^2 \text{Tr} \partial_i U \partial_i U^\dagger \right) - \frac{1}{2} \chi \text{Re} M^\dagger U + \dots \quad (35)$$

where  $v_\pi$  is the pion velocity,  $M$  is the mass matrix introduced as an external field,  $U$  is the chiral field and the covariant derivative  $\nabla_0 U$  is given by  $\nabla_0 U = \partial_0 U - \frac{i}{2} \mu_A (\tau_3 U + U \tau_3)$  with  $\mu_A$  the axial isospin chemical potential. The ellipsis stands for higher order terms in spatial derivatives and covariant derivatives.

The quantities that we need to study are the vector isospin susceptibility (VSUS)  $\chi_V$  and the axial-vector isospin susceptibility (ASUS)  $\chi_A$  defined in terms of the vector charge density  $J_a^0(x)$  and the axial-vector charge density  $J_{5a}^0(x)$  by the Euclidean correlators:

$$\delta_{ab} \chi_V = \int_0^{1/T} d\tau \int d^3 \vec{x} \langle J_a^0(\tau, \vec{x}) J_b^0(0, \vec{0}) \rangle_\beta, \quad (36)$$

$$\delta_{ab} \chi_A = \int_0^{1/T} d\tau \int d^3 \vec{x} \langle J_{5a}^0(\tau, \vec{x}) J_{5b}^0(0, \vec{0}) \rangle_\beta \quad (37)$$

where  $\langle \rangle_\beta$  denotes thermal average and

$$J_a^0 \equiv \bar{\psi} \gamma^0 \frac{\tau^a}{2} \psi, \quad J_{5a}^0 \equiv \bar{\psi} \gamma^0 \gamma^5 \frac{\tau^a}{2} \psi \quad (38)$$

where  $\psi$  is the quark field and  $\tau^a$  is the Pauli matrix generator of the flavor  $SU(2)$ . Given the effective action described by (35), with possible non-local terms ignored, the axial susceptibility (ASUS) takes the simple form

$$\chi_A = - \frac{\partial^2}{\partial \mu_A^2} L_{eff} |_{\mu_A=0} = f_\pi^2. \quad (39)$$

The principal point to note here is that *as long as the effective action is given by local terms (subsumed in the ellipsis) involving the  $U$  field, this is the*

*whole story:* There is no contribution to the ASUS other than the temporal component of the pion decay constant.

Next one assumes that at the chiral phase transition point  $T = T_c$ , the restoration of chiral symmetry dictates the equality

$$\chi_A = \chi_V. \quad (40)$$

While there is no lattice information on  $\chi_A$ ,  $\chi_V$  has been measured as a function of temperature [53,54]. In particular, it is established that

$$\chi_V|_{T=T_c} \neq 0, \quad (41)$$

which leads to the conclusion that

$$f_\pi^t|_{T=T_c} \neq 0. \quad (42)$$

On the other hand, it is expected and verified by lattice simulations that the space component of the pion decay constant  $f_\pi^s$  should vanish at  $T = T_c$ . One therefore arrives at

$$v_\pi^2 \sim f_\pi^s/f_\pi^t \rightarrow 0, \quad T \rightarrow T_c. \quad (43)$$

This is the main conclusion of the standard chiral theory [55].

What this means physically is as follows. The pole mass of the pion  $m_\pi^p$  in a heat bath is related to the screening mass  $m_\pi^s$  via  $m_\pi^{p^2} = v_\pi^2(m_\pi^{s^2} + \vec{k}^2)$ . Thus the vanishing of the pole mass would imply in this scenario the vanishing of the pion velocity. In some sense this result would indicate a maximal violation of Lorentz invariance and this will be at a stark variance with what we find in HLS/VM theory described below.

This elegant argument has a caveat. If one uses the same argument for the VSUS, one gets a wrong answer. The effective Lagrangian for calculating the vector correlators is of the same form as the ASUS, Eq. (35), except that the covariant derivative is now defined with the vector isospin chemical potential  $\mu_V$  as  $\nabla_0 U = \partial_0 U - \frac{1}{2}\mu_V(\tau_3 U - U\tau_3)$ . Now if one assumes as done above for  $\chi_A$  that possible non-local terms can be dropped, then the VSUS is given by

$$\chi_V = -\frac{\partial^2}{\partial \mu_V^2} L_{eff}|_{\mu_V=0} \quad (44)$$

which can be easily evaluated from the Lagrangian. One finds that

$$\chi_V = 0 \tag{45}$$

for all temperatures. While it is expected to be zero at  $T = 0$ , the vanishing of  $\chi_V$  for  $T \neq 0$  is at variance with the lattice data at  $T = T_c$ .

The sigma model prediction (43) can be simply understood from the fact that in the absence of other degrees of freedom,  $\chi_A$  is directly related to  $f_\pi^t$  and  $\chi_A$  is equal to  $\chi_V$  at the chiral restoration point. Since  $\chi_V$  is seen to be nonzero,  $f_\pi^t$  does not vanish at  $T_c$  whereas the space component  $f_\pi^s$  does. Now one can ask why  $\chi_A$  should be given entirely by  $f_\pi^t$  at  $T_c$ . There is no reason why there should not be some additional contributions to  $\chi_A$  other than from  $f_\pi^t$ . Indeed, this is the defect of the pion-only sigma model scenario. We will see below that when the  $\rho$  meson goes massless at  $T_c$ , the longitudinal component of the  $\rho$  meson contributes to  $\chi_A$  on the same footing as the  $\pi$  and hence the observation that the non-vanishing of  $\chi_V$  implies non-vanishing of  $f_\pi^t$  is invalidated.

- *The HLS/VM scenario* [56]

In the presence of the  $\rho$  meson in HLS, it comes out that  $(f_\pi^t, f_\pi^s) \rightarrow (0, 0)$  and  $\chi_A \rightarrow \chi_V \neq 0$  as  $T \rightarrow T_c$ . For this we should start with (27) and work with broken Lorentz invariance. This means that we have to consider the condensates like  $\bar{q}\gamma_\mu D_\nu q$  in the current correlators. It turns out however that in HLS/VM theory such invariance breaking appears as a small correction compared with the main term of  $(1 + \frac{3(N_c^2-1)}{8N_c} \frac{\alpha_s}{\pi})$  in the Lorentz-invariant matching condition of the form

$$\begin{aligned} \frac{F_\pi^2(\Lambda)}{\Lambda^2} = \frac{1}{8\pi^2} \left( \frac{N_c}{3} \right) & \left[ 1 + \frac{3(N_c^2-1)}{8N_c} \frac{\alpha_s}{\pi} + \frac{2\pi^2}{N_c} \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle}{\Lambda^4} \right. \\ & \left. + \frac{288\pi(N_c^2-1)}{N_c^3} \left( \frac{1}{2} + \frac{1}{3N_c} \right) \frac{\alpha_s \langle \bar{q}q \rangle^2}{\Lambda^6} \right]. \end{aligned} \tag{46}$$

This implies that the difference between  $F_{\pi,\text{bare}}^t$  and  $F_{\pi,\text{bare}}^s$  is small compared with their own values, or equivalently, the bare  $\pi$  velocity defined by  $V_{\pi,\text{bare}}^2 \equiv F_{\pi,\text{bare}}^s/F_{\pi,\text{bare}}^t$  is close to one. We will give an estimate of the correction to this bare pion velocity later.

Now given the result that  $V_{\pi,\text{bare}} = 1$ , we need to compute the quantum corrections so as to compare with nature. Here Sasaki's non-renormalization theorem [52] will help. The argument for the theorem goes as follows. At  $T \ll T_c$ , the pion velocity – denoted  $v_\pi$  for the physical quantity – receives a hadronic thermal correction from the pion field of the form

$$v_\pi^2(T) \simeq V_\pi^2 - N_f \frac{2\pi^2}{15} \frac{T^4}{(F_\pi^t)^2 M_\rho^2} \quad \text{for } T < T_c. \quad (47)$$

Here the longitudinal component of the  $\rho$  field (called  $\sigma$  in Harada-Yamawaki theory) is suppressed by the Boltzmann factor  $\exp[-M_\rho/T]$ , and hence only the pion loop contributes to the pion velocity. Now approach  $T_c$ . Then the vector meson mass drops toward zero due to the VM and the Boltzmann factor  $\exp[-M_\rho/T]$  is no longer a suppression factor. Thus at tree order, the contribution from the longitudinal vector meson ( $\sigma$ ) exactly cancels the pion contribution. Similarly the quantum correction generated from the pion loop is exactly canceled by that from the  $\sigma$  loop. Accordingly we conclude

$$v_\pi(T) = V_{\pi,\text{bare}}(T) \quad \text{for } T \rightarrow T_c. \quad (48)$$

In sum, the pion velocity in the limit  $T \rightarrow T_c$  is protected by the VM against both quantum and hadronic loop corrections at one loop order [52]. This implies that  $(g_L, a^t, a^s, V_\pi) = (0, 1, 1, \text{any})$  forms a fixed line for four RGEs of  $g_L, a^t, a^s$  and  $V_\pi$ . When a point on this fixed line is selected through the matching procedure (this is explained in detail in [56]), that is to say that when the value of  $V_{\pi,\text{bare}}$  is fixed, the present result implies that the point does not move in a subspace of the parameters. Approaching the chiral symmetry restoration point, the physical pion velocity itself will flow into the fixed point.

The corrections due to the breaking of Lorentz invariance to the bare pion velocity are model-dependent and cannot be pinned down accurately. However a rough estimate shows that they are small. With a wide range of QCD parameters,  $\Lambda_M = 0.8 - 1.1 \text{ GeV}$ ,  $\Lambda_{QCD} = 0.30 - 0.45 \text{ GeV}$  and the range of critical temperature  $T_c = 0.15 - 0.20 \text{ GeV}$ , it has been found that

$$\delta_{\text{bare}}(T_c) = 0.0061 - 0.29. \quad (49)$$

Thus we find the *bare* pion velocity to be close to the speed of light:

$$V_{\pi,\text{bare}}(T_c) = 0.83 - 0.99. \quad (50)$$

Now thanks to the non-renormalization theorem [52], which is applicable here as well, i.e.,  $v_\pi(T_c) = V_{\pi,\text{bare}}(T_c)$ , we arrive at the physical pion velocity at chiral restoration:

$$v_\pi(T_c) = 0.83 - 0.99. \quad (51)$$

The dramatic difference in predictions for  $v_\pi$  near  $T_c$  between the sigma model scenario,  $v_\pi \sim 0$  and the HLS/VM scenario,  $v_\pi \sim 1$  should be testable by

experiments or lattice calculations. In fact this issue has been addressed in connection with the STAR data on HBT [57]. However the result is inconclusive.

## 9 Other Observables

### 9.1 Relation of Brown-Rho scaling to Harada-Yamawaki vector manifestation

Brown-Rho scaling [1] was one of the first attempts in nuclear physics to formulate medium dependent effects associated with the approach to chiral restoration as the scale, either with temperature or density or with both, was increased. A simple way to see that dynamically generated masses do scale was introduced by Lutz et al. [58] through the Gell-Mann, Oakes, Renner relation

$$f_\pi^2 m_\pi^2 = 2\bar{m}\langle\bar{q}q\rangle, \quad (52)$$

where  $\bar{m}$  is the bare quark mass. Both  $\bar{m}$  and  $m_\pi$ , which is protected against scaling to the extent that it is a Goldstone boson, do not scale. This relation would then produce

$$\frac{f_\pi^*}{f_\pi} = \frac{\sqrt{\langle\bar{q}q\rangle^*}}{\sqrt{\langle\bar{q}q\rangle}}, \quad (53)$$

which holds quite well for low densities<sup>9</sup>. In fact, for low densities one has the relation [59,60]

$$\frac{\langle\bar{q}q\rangle^*}{\langle\bar{q}q\rangle} = 1 - \frac{\sigma_\pi n}{f_\pi^2 m_\pi^2} \quad (54)$$

where  $n$  is the vector density. Eq. (54) holds to linear approximation. At higher densities Koch and Brown [61] showed that the entropy from reduced mass hadrons fit the entropy from LGS if one had “Nambu scaling”

$$\frac{m_H^*}{m_H} = \frac{\langle\bar{q}q\rangle^*}{\langle\bar{q}q\rangle}; \quad (55)$$

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<sup>9</sup> To be precise, this relation in medium is a relation for the space component of the pion decay constant which is different from the time component since Lorentz invariance is broken.

i.e., the hadron *in-medium* mass scaled linearly with the quark scalar density. This scaling seems to come out in a number of QCD sum rule calculations, also. It holds in the Harada and Yamawaki RG theory for high temperatures or densities approaching chiral restoration which takes place at the fixed point where  $m_V^*$  and  $g_V^*$  go to zero.

The above applies to what we call the parametric scaling; i.e., to the scaling of the parameters  $F_\pi$ , etc. which enter into the chiral Lagrangian. One must then take this Lagrangian and calculate thermal or dense loops, which will somewhat change the medium dependence. A point which is generally unappreciated in the heavy-ion theory community is that in a heat bath even at low temperatures the (second) loop corrections are mandatory for consistency with the symmetry of QCD. In fact, in the combination of parametric and loop terms, the pole mass of the vector meson increases proportional to  $T^4$  near zero temperature with no  $T^2$  term present as required by the low-energy theorem [62]. As the temperature of chiral restoration  $T_{\chi SR}$  is approached, both the bare mass term and the loop corrections go to zero as  $\langle \bar{q}q \rangle \rightarrow 0$ . In this case the pole mass does directly reflect on chiral structure as does Brown-Rho scaling. *Only in the vicinity of  $T_c$  does BR scaling manifest itself transparently in the pole mass of the vector meson in a heat bath.*

Evaluation of  $g_\pi^*/f_\pi$  with Eqs. (53) & (54) gives a 20% drop in this quantity by nuclear matter density  $n_0$ . This agrees with the value extracted at tree order from pionic atoms [63]. The same decrease is implied by Brown-Rho scaling for  $m_\rho^*$ . However, the dense loop enters also here and, although small, will increase the mass a few MeV. Thus, the decrease of  $\sim 15\%$  in  $m_\rho^*$  by nuclear matter density seems reasonable.

Harada and Yamawaki find that  $m_\rho^*$  scales linearly with  $\langle \bar{q}q \rangle$  as  $m_\rho^* \rightarrow 0$  at the fixed point of chiral symmetry restoration. In fact, although the comparison with lattice results on the entropy is relatively crude in Koch and Brown [61], it is seen that with temperature the scaling of the masses may begin less rapidly than the scaling with  $\langle \bar{q}q \rangle^*$ , but that it quickly becomes as rapid. Brown and Rho [4] found that up to nuclear matter density  $n_0$ ,  $g$  did not scale, but slightly above  $n_0$  the ratio  $g^*/m_\rho^*$  was roughly constant. The ratio is constant going toward the fixed point of Harada and Yamawaki. Thus we believe that the decrease of  $m_\rho^*$  as  $\sqrt{\langle \bar{q}q \rangle^*}$  goes only up to  $n \sim n_0$  and that it then scales linearly with  $\langle \bar{q}q \rangle^*$ . If it decreases  $\sim 20\%$  in going from  $n = 0$  to  $n_0$ , it will then increase  $\sim 2\sqrt{2}$  in going from  $n_0$  to  $2n_0$ , and  $m_\rho^*$  will go to zero at  $n \sim 4n_0$ , the scalar density at chiral restoration. From this estimate we believe

$$n_{\chi SR} \sim 4n_0. \tag{56}$$

Given the Walecka mean field theory [64] and the study of the density and temperature dependence of a system of constituent quarks in the Nambu-Jona-Lasinio theory [17], Brown-Rho scaling appeared quite natural, at least the scaling with density, even a long time before its acceptance (it is not universally accepted even now, although it has come to life rather quickly after each of its many reported deaths).

The Walecka theory showed that the nucleon effective mass decreased with density. Perhaps most convincing of the arguments in its favor was that the spin-orbit term, which depends on  $(m_N^*)^{-2}$  was increased enough to fit experiment. The usual nonrelativistic theories were typically a factor of 2 too low in spin-orbit interaction at that time. What could be more natural than as a nucleon dissolves into its constituents, the masses  $m_Q^*$  of the constituent quarks decreases at the same rate as the nucleon mass  $m_N^*$  in Walecka theory? In fact, this is what happens in the Harada-Yamawaki theory, although it does not contain nucleons (the effect of fermions was studied in [65] by introducing constituent quarks). Once the density is high enough so that constituent quarks become the relevant variables, we should go over to a quark description, as described above and as Bernard et al. [17] did. Then the constituent quark mass will change with increasing density, going to zero the way the constituent quark went over to a current quark as the temperature increased from  $T = 125$  MeV to 175 MeV ( $T_c(\text{unquenched})$ ). Since at zero density nucleons are the relevant variables, it will take some time in adding nucleons in positive energy states before these cancel enough of the condensate of nucleons in negative energy states so that they can go over into loosely bound constituent quarks.

In fact, with a cut off of  $\Lambda = 700$  MeV, close to what we use, Bernard et al. [17] found that in the chiral limit the quark mass went to zero at  $2n_0$ . In our scenario of chiral restoration at  $n \sim 4n_0$  outlined earlier, this then means that  $n$  has to increase from 0 to  $\sim 2n_0$  before the nucleons dissolve into constituent quarks.

## 9.2 Landau Fermi-liquid fixed point and Brown-Rho scaling

The meaning of Brown-Rho scaling has often been misinterpreted in the literature for processes probing densities in the vicinity of nuclear matter density, most recently in connection with the NA60 dilepton data. We wish to clarify the situation by emphasizing the intricacy involved in what the scaling relation represents in the strong interactions that take place in many-nucleon systems. This aspect has been discussed in several previous publications by two of the authors (GEB and MR), but it is perhaps not superfluous to do so once more in view of certain recent developments. What we would like to discuss here is the

connection between the Brown-Rho scaling factor  $\Phi(n)$  (to be defined below) and the Landau parameter  $F_1$  which figures in quasiparticle interactions in Fermi liquid theory of nuclear matter. This discussion illustrates clearly that Brown-Rho scaling cannot *simply* be taken to be *only* the mass scaling as a function of density and/or temperature as is often done in the field. What this illustrates is that the  $\Phi$ , related in an intricate way to a quasiparticle interaction parameter in Landau Fermi-liquid theory of nuclear matter, incorporates not just the “intrinsic density dependence” (IDD in short) associated with Wilsonian matching to QCD, a crucial element of HLS/VM, but also some of what is conventionally considered as many-body interactions near the Fermi surface associated with the Fermi liquid fixed point. It clearly shows that it is dangerous to naively or blindly apply Brown-Rho scaling to such heavy-ion processes as low-mass dileptons where the density probed is not much higher than nuclear matter density, as was done by several workers in QM2005. We will present arguments more specific to dilepton processes in subsection 9.4, but what we mean will already be clear at the end of this subsection.

### 9.2.1 Chiral Fermi liquid field theory (CFLFT)

It was argued by Brown and Rho in [66] (where previous references are given) that addressing nuclear matter from the point of view of effective field theory involves “double decimation” in the renormalization-group sense. The first involves going from a chiral scale or the matching scale  $\Lambda_M$  with “bare Lagrangian” to the Fermi surface scale  $\Lambda_{FS}$  (which will be identified later with  $\Lambda_{low-k}$ ). We shall describe below a recent work [67] that arrives at this result microscopically where the  $\Lambda_{FS}$  will be identified with  $\Lambda_{low-k}$ . How this could be achieved was discussed in a general context by Lynn some years ago [68] who made the conjecture that the Fermi surface could arise from effective field theories as chiral liquid soliton. For the moment, we will simply assume that such a chiral liquid can be obtained. To proceed from there, we exploit three observations (or, perhaps more appropriately, conjectures). First we learn from the work of Shankar [69] that given an effective Lagrangian built around the Fermi surface, decimating fluctuations toward the Fermi surface leads to the “Fermi liquid fixed point” with the quasiparticle mass  $m^*$  and quasiparticle interactions  $F$  being the fixed point parameters. We next learn from Matsui’s argument [70] that Walecka mean field theory is *equivalent* to Landau Fermi liquid theory. The third observation is that Walecka mean-field theory can be obtained in the mean field of an effective chiral Lagrangian in which (vector and scalar) massive degrees of freedom are present, or equivalently, an effective chiral Lagrangian with higher-dimension operators (such as four-Fermi operators) [71,72,73]. Friman and Rho [74] combined the above three to write an effective chiral Lagrangian endowed with Brown-Rho scaling that in mean field gives *Landau Fermi-liquid theory at the fixed point that is consistent with chiral symmetry*. We call this “chiral Fermi liquid field theory (CFLFT)” to



distinguish it from the microscopic theory of Holt et al. [67].

As reviewed in [66], there are two classes of effective Lagrangians that should in principle yield the same results in the mean field. One is closely related to a generalized HLS (GHLS) theory where a scalar and nucleons are added to vector mesons. Restricted to symmetric nuclear matter, it has the simple form

$$L_{II} = \bar{N}(i\gamma_\mu(\partial^\mu + ig_v^*\omega^\mu) - M^* + h^*\sigma)N - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{m_\omega^{*2}}{2}\omega^2 - \frac{m_\sigma^{*2}}{2}\sigma^2 + \dots \quad (57)$$

where the ellipsis denotes higher-dimension operators and the star refers to “parametric density dependence” that emerges from a Wilsonian matching to QCD of the type described by Harada and Yamawaki [2]. We have left out (pseudo)Goldstone fields and isovector and strange vector meson fields which do not contribute at mean field level. Note that contrary to its appearance, (57) is actually consistent with chiral symmetry since here both the  $\omega$  and  $\sigma$  fields are *chiral singlets*. In fact, the  $\sigma$  here has nothing to do with the chiral fourth-component scalar field of the linear sigma model except perhaps near the chiral phase transition density where “mended symmetry” may intervene; it is a “dilaton” connected with the trace anomaly of QCD.

An alternative Lagrangian which is in a standard chiral symmetric form involves only the pion and nucleon fields which may be considered as arising when the heavy mesons – both scalar and vector mesons – are integrated out:

$$L_I = \bar{N}[i\gamma_\mu(\partial^\mu + iv^\mu + g_A^*\gamma_5 a^\mu) - M^*]N - \sum_i C_i^*(\bar{N}\Gamma_i N)^2 + \dots \quad (58)$$

where the ellipsis stands for higher dimension and/or higher derivative operators and the  $\Gamma_i$ ’s are Dirac and flavor matrices as well as derivatives consistent with chiral symmetry. Here we reinstated the pionic vector and axial vector fields  $v_\mu$  and  $a_\mu$  respectively, since the pion contributes (through exchange) to the Landau parameters. We will go back and forth between the two Lagrangians in our discussion.

Leaving out the details which can be found in [66,75], we summarize the essential features in what is obtained for nuclear matter. In calculating nuclear matter properties with our effective action, the first thing to do is to determine how the nucleon and meson masses scale near nuclear matter saturation density. This cannot be gotten by theory, so we need empirical information. This can be done by looking at the response of nuclear matter to external fields, i.e., the photon. This was first done in [74,76] using (58) in which the isovector anomalous nuclear orbital gyromagnetic ratio  $\delta g_l$  was expressed in

terms of Brown-Rho scaling plus contributions from the pion to the Landau parameter  $F_1$ <sup>10</sup>

$$\delta g_l = \frac{4}{9}[\Phi^{-1} - 1 - \frac{1}{2}\tilde{F}_1^\pi], \quad (59)$$

where  $\tilde{F}_1^\pi$  is the pionic contribution to the Landau parameter  $F_1$  – which is precisely calculable for any density thanks to chiral symmetry – and

$$\Phi(n) = \frac{m_M^*(n)}{m_M}, \quad (60)$$

which is referred to as the “Brown-Rho scaling factor.” Here the subscript  $M$  stands for the mesons  $M = \sigma, \rho, \omega$ . The isovector gyromagnetic ratio  $\delta g_l$  is measured experimentally. The most precise value comes from giant dipole resonances in heavy nuclei [77]:  $\delta g_l = 0.23 \pm 0.03$ . With  $\frac{1}{3}\tilde{F}_1^\pi = -0.153$  at nuclear matter density  $n_0$ , we get from (59),

$$\Phi(n_0) = 0.78, \quad (61)$$

which is consistent with the value obtained in deeply bound pionic atoms [78]

$$\frac{f_\pi^*(n_0)}{f_\pi} \simeq 0.80. \quad (62)$$

We should stress that this is a value appropriate for normal nuclear matter density which should be reliable near the Fermi liquid fixed point. For describing nuclear matter properties, we need to know how it varies near nuclear matter equilibrium density. A convenient parametrization is

$$\Phi(n) = \frac{1}{1 + yn/n_0} \quad (63)$$

with  $y = 0.28$ .

The Landau effective mass of the quasiparticle at the fixed point is given by

$$m_N^*(n)/m_N = \left( \Phi^{-1} - \frac{1}{3}\tilde{F}_1^\pi \right)^{-1}, \quad (64)$$

which at the equilibrium density predicts

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<sup>10</sup> This relation is valid up to near nuclear matter density, that is, near the Fermi-liquid fixed point and may not be extended to much higher densities.

$$m_N^*(n_0)/m_N = 0.67. \quad (65)$$

Note that the nucleon mass scales slightly faster than meson masses. This was noted in [1] in terms of the scaling of  $g_A$  in medium.

We now look at other properties of nuclear matter with (57). Our construction of chiral Fermi liquid theory instructs us to treat the Lagrangian in mean field with the mass and coupling parameters subject to the Brown-Rho scaling. With the standard free-space values for the  $\omega$  and  $\rho$  mesons and the scalar meson mass  $m_\sigma \approx 700$  MeV<sup>11</sup>, the properties of nuclear matter come out to be [75,79]

$$\begin{aligned} B = 16.1 \text{ MeV}, \quad k_F = 258 \text{ MeV}, \quad K = 259 \text{ MeV}, \\ m_N^*/m_N = 0.67 \end{aligned} \quad (66)$$

where  $B$  is the binding energy,  $k_F$  the equilibrium Fermi momentum and  $K$  the compression modulus. The values (66) should be compared with the standard “empirical values” [80]

$$\begin{aligned} B = 16.0 \pm 0.1 \text{ MeV}, \quad k_F = 256 \pm 2 \text{ MeV}, \quad K = 250 \pm 50 \text{ MeV}, \\ m_N^*/m_N = 0.61 \pm 0.03. \end{aligned} \quad (67)$$

The predicted results (66) are in a good agreement with empirical values. Given the extreme simplicity of the theory, it is rather surprising.

We should remark that what makes the theory particularly sensible is that it is thermodynamically consistent in the sense that both energy and momentum are conserved [75,81]. This is a nontrivial feat. In fact, it has been a major difficulty for nuclear matter models based on Lagrangians with density-dependent parameters to preserve the energy-momentum conservation. In the present theory, this is achieved by incorporating a chiral invariant form for the density operator.

### 9.2.2 *Brown-Rho scaling and microscopic calculation of the Landau parameters*

In the CFLFT description given above, we relied on three observations – the validity of which are yet to be confirmed – on the connection between an effective chiral action (or an effective chiral Lagrangian in mean field) and Landau’s Fermi liquid fixed point theory, in particular with one of the fixed

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<sup>11</sup> Not to be confused with the Goldstone boson  $\sigma$  in HLS theory. Here it is a chiral singlet effective field of scalar quantum number that figures in Walecka-type mean-field theory.

point parameters mapped to Brown-Rho scaling at the corresponding density. In a recent work, Holt et al. [67] obtained successfully a realistic Fermi liquid description of nuclear matter in a microscopic approach that combines the two decimations subsumed in the CFLFT approach [79]. The approach of Holt et al. starts with phenomenological potentials fit to scattering data up to a momentum  $\Lambda_{NN} \sim 2.1 \text{ fm}^{-1}$ .<sup>12</sup> To understand their result, we can recast their argument in terms of the HLS theory that we are using. There is no such potential built from HLS Lagrangian in the literature. However we expect, based on the work of Bogner et al. [82], the resulting driving potential  $V_{low-k}$  to be qualitatively the same for the HLS and phenomenological models for low-energy processes. This, we suggest, is essentially the manifestation of the power of what is called “more effective effective theory” (or MEEFT for short) explained in [66].

Let us imagine that we have a generalized hidden local symmetry (GHLS) theory discussed in section 3.2 that contains a complete set of relevant degrees of freedom for nuclear matter, say,  $\pi$ ,  $\rho$ ,  $\omega$ ,  $a_1$ ,  $\sigma$ , etc., matched to QCD at a matching scale  $\Lambda_M$ . There are no explicit baryon degrees of freedom in this theory. However as discussed in section 3.1, baryons must emerge as skyrmions. Since no description of nuclear dynamics starting from a GHLS exists – and we see no reason why it cannot be done – one can alternatively introduce baryon fields as matter fields and couple them in a hidden local symmetric way. This is what one does in standard chiral perturbation theory with global chiral symmetry with pions and nucleons as the only explicit degrees of freedom. The “bare” Lagrangian obtained by Wilsonian matching will carry such parameters as masses and coupling constants endowed with an “intrinsic” background (temperature or density) dependence. These are the quantities that track the properties of the quark and gluon condensates in medium, and hence Brown-Rho scaling.

Given the “bare” Lagrangian so determined, one can then proceed in three steps:

- (1) First one constructs NN potentials in chiral perturbation theory with the vector mesons treated à la Harada and Yamawaki [2] – here hidden local symmetry plays a crucial role even at zero density as emphasized in [2]. The chiral perturbation procedure is as well formulated in HLS theory as in the standard approach without the vector degrees of freedom.
- (2) Next one performs a (Wilsonian) renormalization-group decimation to the “low-k scale”<sup>13</sup>  $\Lambda_{low-k} \approx 2 \text{ fm}^{-1} \sim \Lambda_{NN}$  to obtain the  $V_{low-k}$ . As stated, we expect the result for  $V_{low-k}$  to be basically the same as that

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<sup>12</sup> This momentum corresponds to the Berkeley relative momentum from the 350 MeV laboratory energy in the cyclotron.

<sup>13</sup> In this scheme, this “low-k scale” corresponds to  $\Lambda_{FS}$  introduced above.

obtained in [82,83] for the T matrix for NN scattering for which matter density is low. However it will differ in medium due to the intrinsic background dependence which is missing in [83]. This step will correctly implement the *first decimation* of [66] not only in free-space but also in dense medium. The intrinsic dependence incorporated at this stage is missing in *all* works found in the literature.

- (3) Finally one feeds the  $V_{low-k}$  so determined into the Bäckman, Brown and Niskanen nonlinear equation (their equation (5.3)) [84], which resulted from the truncation of the Babu-Brown [85] equation in the sum over Fermi Liquid parameters to  $l = 0$  and 1, and solves it by iteration. (For a Green's function formalism for Landau Fermi liquid theory, see [86].) The Babu-Brown equation introduces the induced interaction into dynamical calculations involving Fermi liquid theory and has its own renormalization group treatment [83] which has very successfully been carried out for neutron matter.

These procedures will lead to the Landau parameters given as the sum of a “driving term” and an “induced term.” Given the Landau quasiparticle interactions so determined, the standard Fermi liquid arguments are then applied to computing the energy density, Landau effective mass, compression modulus, etc. that describe nuclear matter. Now had the above three-step procedure been followed with HLS, the theory would have Brown-Rho scaling automatically incorporated. However since Holt et al. [67] take, for step 1, phenomenological potentials in which the intrinsic density dependence (IDD) required by matching to QCD is missing, they need to implement the IDD by hand. They find that without IDD, the known properties of nuclear matter compression modulus, etc. cannot be reproduced correctly. To incorporate Brown-Rho scaling into the potential they employ, e.g., the Bonn one-boson-exchange potential, they introduce  $\sigma$ -tadpole self-energy corrections to the masses of the nucleons and exchanged bosons. Although this procedure may lack the consistency achieved through Wilsonian matching, it should however be equivalent to Brown-Rho scaling in its simplest form. With the Brown-Rho scaling suitably implemented, the result obtained in [67] comes out to be quite satisfactory.

There are several observations one can make from this result. 1) One can think of this as a confirmation of the soundness of the double decimation procedure. 2) Both the microscopic approach [67] and the effective theory approach [79] – which complement each other – indicate the importance of Brown-Rho scaling in the structure of nuclear matter. 3) There must be a relation – most likely quite complicated – between  $\Phi$  and the microscopic potential  $V_{low-k}$  valid at nuclear matter density. We believe this relation to result from the scale invariance in nuclear phenomena which results when pion exchange is unimportant in nuclear phenomena, as reviewed in Brown and Rho [66]. There are very few places in *nuclear spectra* where the pion plays the

main role<sup>14</sup>. For instance, it does not play much of a role in the polarization phenomena reviewed by Brown and Rho [66], so they come out to be in some sense “scale invariant” for low densities. The pion is, of course, protected from mass change by chiral invariance, and, therefore, does not participate in this “scale invariance.” However, in the second-order tensor interaction which is of primary importance for the saturation of nuclear matter, the  $\pi$  and  $\sigma$  play counterpoint; they enter incoherently, their coupling having opposite sign. The dropping of the  $\rho$ -mass in Brown-Rho scaling therefore cuts down the tensor interaction greatly, running up the compression modulus substantially. In the usual calculations which do not employ the intrinsic (*in-medium*) dependence of the masses on density, this effect is included empirically as a three-body interaction. This seems to be the only place in nuclear spectra where Brown-Rho scaling seems to be really needed *explicitly* in the nuclear many-body problem, although Holt et al. [67] find it helpful to include Brown-Rho scaling in terms of a scalar tadpole in order to bring the effective mass  $m_N^*$  down towards the lower Walecka values. Nuclear physics have, however, lived with a substantial spread in effective masses – which cannot be measured directly in nuclear physics – for years, so what we consider as an improvement in  $m_N^*$  is not universally accepted. This reflects again on an intricate interplay between Brown-Rho scaling and many-body interactions.

As a final remark in this subsection, let us return to the implications of the manifestation of chiral symmetry in dense (or hot) medium. It is clear that the connection between Brown-Rho scaling and many-body interactions is highly intricate, particularly near nuclear matter density, and needs to be carefully assessed case by case. As we learned from HLS with the vector manifestation fixed point, Brown-Rho scaling can serve as a clean-cut litmus for chiral restoration – a matter of intense current interest in heavy ion physics – only near the critical point. Only very close to the critical point is the scaling factor  $\Phi$  directly locked to the chiral order parameter  $\langle \bar{q}q \rangle^*$ . Far away from that point, particularly near normal nuclear matter density, the connection, strongly infested with many-body interactions, can be tenuous at best. To the extent that the dileptons in NA60 as well as in CERES, for instance, do not selectively sample the state of matter near the chiral transition point, information on the order parameter cannot be extracted cleanly from the measured spectral function.

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<sup>14</sup> One should, however, note that this is not the case in nuclear response functions, namely in nuclear matrix elements of electroweak currents. It is known that in certain transition matrix elements, such as in M1 transitions and axial charge transitions, soft-pions play an extremely important role. This is referred to as “chiral filter mechanism [87].”

### 9.3 *Fluctuating around the VM fixed point*

The traditional approach to hadronic physics in (hot and/or dense) medium has been to start with a Lagrangian with appropriate symmetries built on the matter-free vacuum and incorporate medium effects mostly at low orders in perturbation theory. As stated at several places in our work, while this can be a valid procedure when the system is near  $T = n = 0$ , it is not at all clear how such low-order calculations that do not account for the intrinsic background dependence can be trusted when the temperature and/or density is near the critical point. Now in the framework of hidden local symmetry theory with the VM, there is another point from which one can do calculations accurately and systematically, namely near the VM fixed point. The strategy of fluctuating around the VM fixed point was discussed in some detail in the Nagoya lecture [88]. As discussed there, even certain properties of hadrons in matter-free space that highlight chiral symmetry – such as for instance the chiral doubling in heavy-light hadrons – can be more readily treated if started from the VM. It is, however, for processes that take place near the chiral restoration point that fluctuating around the VM fixed point will prove to be a lot more powerful and efficient. One specific example is the electron-driven kaon condensation in dense stellar matter [89].

#### 9.3.1 *Kaon condensation treated from the VM fixed point*

Kaon condensation treated with a theory constructed around the  $T = n = 0$  vacuum, much discussed in the literature, is beset with many complications associated with the strong coupling involved in the interactions. Here we briefly describe how we can avoid the plethora of complications and zero-in on the main mechanism of kaon condensation when treated from the vector manifestation fixed point.

What is the problem in starting from the  $T = n = 0$  vacuum? Here one typically does chiral perturbation theory with a chiral Lagrangian defined for elementary interactions around the vacuum. Several complications arise in doing so. The first is the problem of identifying the relevant degrees of freedom and deciding how to treat them. For instance,  $\Lambda(1405)$  plays an important role in the anti-kaon–nucleon interaction near threshold [90] and hence should be important when low-energy kaon-nucleon interactions are to be taken into account. Depending upon the energy involved near the threshold, the kaon-nucleon interaction could be repulsive or attractive. This would imply that one would have to do multichannel calculations to correctly account for the interactions as density moves up from zero. Higher order chiral perturbation terms can become very important which means that one would have to face a theory in which there can be a large number of undetermined (free) parameters,

with the attendant loss of predictive power. Various mechanisms that cannot be accounted for in low order chiral perturbation series, such as short-range correlations between nucleons [91], can have drastic effects on determining the critical density at which kaons can condense. At present, there is no systematic way to assess whether these effects survive high-order treatments consistent with the premises of QCD. We suggested in [92] that these mechanisms are “irrelevant” in the RGE sense in the density regime where kaons condense<sup>15</sup>. Assuming that kaons condense in the vicinity of the chiral restoration point, which is the vector manifestation fixed point, it was suggested to be more expedient to start from the VM fixed point to locate the kaon condensation point.

In doing this, there is a subtle point that needs to be attended to. Seen bottom-up, once kaons condense, the state is no longer the usual Fermi liquid making up nuclear matter. As discussed in [94], the Fermi seas of the up and down quarks get distorted from that of a Fermi liquid with, among others, the isospin symmetry and the parity spontaneously broken due to the condensate  $\langle K \rangle$  etc., and it is not clear whether and how this state makes a transition to the Wigner mode in which chiral symmetry is restored. This conundrum is avoided in the kaon condensation scenario suggested in [95], which we believe is the scenario chosen by compact stars. In this scenario, electrons with high chemical potential, not directly involved in the strong interaction that leads to the flow towards the VM fixed point, induce the “crash” by decaying into kaons at a density below that of the chiral phase transition. The RGE flow up to the crash point is dictated by HLS dynamics and hence “knows” about the VM fixed point and its location in density, assuming that the electrons do not distort the strong interactions. Were it not for the electrons, the HLS/VM would most likely be irrelevant since the flow à la HLS/VM would be stopped when kaons condense.

Seen from the HLS/VM point, the only relevant interactions between kaons and fermions – which would be quasiquarks instead of baryons – would be via exchanges of the light vector mesons,  $\rho$  and  $\omega$ <sup>16</sup>. With Brown-Rho scaling

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<sup>15</sup> An example is the four-Fermi interaction that involves  $\Lambda(1405)$  which we might classify as “dangerously irrelevant” in the sense used in condensed matter systems [93]: Certain high dimension perturbations are found to be important in the paramagnet phase but irrelevant at the critical point. In the same vein, the four-Fermi interaction important near the  $KN$  threshold is important for triggering kaon condensation but highly irrelevant for determining the critical density.

<sup>16</sup> In HLS/VM, the  $a_1$ ’s are joined by a scalar  $s$  at the critical point. One would expect from Weinberg’s mended symmetry [96] that these degrees of freedom will also become massless in the chiral limit at the critical point as indicated in GHLS [6,7], so the question could be raised as to how these degrees of freedom would enter in the process of kaon condensation. We have no clear answer to this question at the moment.



taken into account, the prediction based on the VM fixed point at  $n_{\chi SR} \sim 4n_0$  is that kaons will condense at [92]

$$n_K \sim 3n_0. \quad (68)$$

The mass of the neutron star in J0751+1807 has been measured by Nice et al. [97]. At 95% confidence the mass is  $2.1^{+0.4}_{-0.5} M_\odot$ . This causes a problem for our  $n_K \sim 3n_0$  scenario for the maximum neutron star mass of  $1.5 M_\odot$ . Lee, Brown and Park [98] indicated that probably some repulsion between  $K^-$ -mesons in the condensate after the neutron star is born should be introduced, in order to take into account the short-range repulsion between the  $K^-$ -mesons which are made up out of fermions – the quarks – and therefore should experience some van der Waals type repulsion at short distances. Since the  $K^-$  mesons are small in extent and relatively diffuse, this repulsion would not be expected to be large. Lee, Brown and Park [98] show that the maximum neutron star mass could be raised to  $1.7 M_\odot$  without upsetting the pattern of well-measured relativistic neutron star binaries, but with a higher stable mass for neutron stars, most binaries would end up with pulsar mass several tenths of a solar mass greater than the companion mass, which is not seen in the binaries.

#### 9.4 Dilepton production

In Quark Matter 2005 (proceedings to be published in Nuclear Physics) the calculations of Ralf Rapp in what Rapp interpreted as various different scenarios such as dropping masses, enhanced widths due to many-body nuclear interactions, etc. were compared with the dilepton data of the collaboration NA60, with the conclusion that the dropping mass scenario was disproved. We claim that to conclude, therefore, that Brown-Rho scaling is also disproved is totally wrong. As discussed above in section 9.2, in the way formulated here, Brown-Rho scaling *near nuclear matter density* contains a variety of different aspects of nonperturbative QCD dynamics consisting not only of intrinsic background dependence characterized by the quark condensate – the order parameter – but also of certain aspects of many-body dynamics associated with the Fermi surface (in the case of dense medium). As mentioned before, the situation is vastly clearer and more transparent in the HLS framework *only near* the critical point. Thus measurements that sample mainly the vicinity of normal nuclear matter density cannot be directly related to effects signalling partial chiral symmetry restoration. Our assertion here will be that dilepton measurements so far performed, including the NA60, have not probed the regime where the signal can unambiguously be seen. It would be premature to make a conclusion regarding the validity of Brown-Rho scaling based on the predictions of a naive dropping mass scenario.

The hope of the dilepton experiments of the CERES and NA60 type was to probe the chiral symmetry structure of the vacuum as temperature and/or density is varied. What transpires clearly from the NA60 measurement is that the spectral function in the vector meson channel (or in fact any channel) involves a plethora of assumptions whose validity is not fully under control, compounded with a multitude of different reaction mechanisms that are difficult to separate. Among others, even assuming thermal equilibrium which justifies the notion of temperature, how the system produced at high temperature and/or density evolves in the expansion until the dileptons are detected must play a crucial role but is not at present fully controlled. The measured quantities at a given temperature, density, etc. contain a mixture of various parameters that are not directly connected to the intrinsic QCD properties that one wants to extract. This means that in order to address the purported issue of chiral symmetry, one would have to have a systematic and consistent theoretical tool to sort out all the elements that enter in the analysis of the experiment. Furthermore, one needs to incorporate the experimental conditions, such as the cuts, etc. before one can confront the data with theoretical predictions.

We do not have the necessary codes to make calculations that can be compared with the data. But we know what theoretical ingredients have to be included in the framework we have developed. It has been pointed out [99] that the minimum ingredients for testing Brown-Rho scaling in *any* process are three-fold: (1) the subtle interplay between the intrinsic background dependence in the parameters of the HLS Lagrangian, which follows from matching EFT to QCD, and quantum loop corrections with that Lagrangian, (2) the essential features of the vector manifestation, e.g., the violation of the vector dominance assumption as temperature and/or density approaches the critical point, and (3) the many-body nature of the system with Fermi surfaces that is inherent in the HLS Lagrangian in medium, e.g., the “fusing” of Brown-Rho scaling and “sobar” configurations [100] that arise from particle-hole excitations. All of these elements are intricately connected. *Given the intricacy involved – which we admit is not easy to unravel fully – our conclusion is that dilepton processes in the presently available kinematics of heavy-ion collisions are not – contrary to what has been believed – a clean snapshot for chiral properties of hadrons in medium.*

We give a brief summary of these different ingredients. The last effect is approximately accounted for but the first two are absent in the theoretical calculations presented at QM 2005.

- (1) *Parametric background dependence*

At present, the systematic temperature dependence or density dependence of the parameters of the Lagrangian, such as masses and gauge coupling con-

stants, that is required by matching HLS to QCD is not known reliably except very near the  $T = n = 0$  “vacuum” and the VM fixed point (the same consideration holds in GHLS theory). Thus the temperature and/or density dependence consistent with HLS/VM of the spectral functions in the pertinent channels is not available for analyzing reliably the NA60 data. However as mentioned above, we can make a good guess from lattice results as to how they interpolate between the two end points for the case of temperature and from nuclear phenomenology for the case of density. To summarize, in terms of the temperature, the gauge coupling  $g$  that controls all other parameters of the Lagrangian changes little between  $T = 0$  and the flash temperature  $T_{flash} \sim 125$  MeV, at which point the soft glue starts melting and then drops roughly linearly in the quark condensate to zero (in the chiral limit) at  $T_c \sim 175$  MeV. With Harada-Sasaki’s thermal loop corrections [101], *indispensable for satisfying low-energy theorems*, one can estimate roughly that the  $\rho$  mass does not move much from its free-space value up to  $T_{flash}$ , after which it drops to zero proportional to  $g$  at  $T_c$ . With density, it has been shown (see [4] for a review) that  $g$  does not change appreciably up to normal nuclear density  $n_0$  and starts dropping linearly in quark condensate slightly above  $\sim 2n_0$  (the precise value is neither known nor very important for our purpose).

This parametric dependence is missing in *all* of the analyses of dilepton data available in the literature. This is a serious defect if one wants to study chiral symmetry in dilepton production in heavy-ion collisions.

- (2) *Vector dominance violation*

As shown by Harada and Sasaki [101], vector dominance can be badly broken near the critical temperature. As mentioned in [99], in dense medium the parameter  $a$  in HLS theory tends to quickly approach 1<sup>17</sup>. This means that density also breaks vector dominance. To take this VD violation into account, we need to determine which fixed point of the three uncovered in GHLS theory is arrived at by temperature at chiral restoration. We will consider the VM and Hybrid types here in which vector dominance is seriously violated.

The net effect is that the photon coupling in the dimuon production could be cut down by a factor as large as 4. This effect is also missing in *all* presently available theoretical works on dilepton production.

- (3) *“Sobar” configurations*

The presence of the Fermi surface in many-nucleon systems requires that in addition to the elementary mesonic excitations of the  $\rho$  (and  $\omega$ ) quantum numbers, there can be collective particle-hole excitations of the same quan-

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<sup>17</sup> This point was discussed in a more general setting in [88].

tum numbers. This means specifically that the spectral function in the  $\rho$  channel, say, will receive contributions from, among others, both the elementary  $\rho$  meson and the “rhobar” (e.g.,  $N^*(1520)N^{-1}$ ) modes. In a Lagrangian description, such modes can be included perturbatively given a Lagrangian that explicitly contains the relevant excited baryons. However, constructing such a Lagrangian fully consistent with QCD is a daunting task that has not yet been achieved. So far what has been done is a phenomenological Lagrangian calculation, that is, at tree order. This is the Rapp-Wambach description [100]. This may be justified in very dilute systems but is highly suspect in dense medium since the two effects (1) and (2) mentioned above, which we consider to be crucial for the issue of chiral symmetry structure, are missing. As explained in [2,88], without local gauge symmetry, it is not obvious how to consistently account for them in that approach.

The most satisfactory approach to account for this many-body aspect would be to work directly with the HLS Lagrangian consisting only of mesonic fields, perhaps with an infinite tower, with baryons appearing as skyrmions. Unfortunately, as mentioned, skyrmions with integer baryon numbers are problematic near the chiral phase transition, since the relevant fermionic degrees of freedom are more likely quasiquarks subject to Brown-Rho scaling. As mentioned, the difficulty is that there seem to be no stable quasiquarks, that is, qualitons. An alternative – and more workable – approach would be, as sketched in [102], to take into account the presence of the Fermi sea by introducing “sobar” configurations. This would correspond to a bosonization of particle-hole configurations in a spirit similar to the bosonization of the Landau Fermi liquid system [103].

It remains to be seen how a consistent HLS calculation taking into account the above ingredients combined with thermal/dense loop corrections fares with the dilepton data. Such a calculation – and only such a calculation – will carry well-defined and meaningful information on the chiral symmetry property of the matter. In the present situation, we feel that the best one can do is to “fuse” the two configurations (“elementary” and “sobar”) as reviewed in [4], with Brown-Rho scaling taking into account both effects in an approximate way.

## 10 Conclusions

In this review, an argument is presented that Harada-Yamawaki hidden local symmetry theory emerges naturally as a truncation of an infinite tower of hidden local fields present in the holographic dual approach to QCD. Restricted to the lowest member of the infinite tower and Wilsonian matched to QCD (Yang-Mills gauge theory with fundamental quark fields), the resulting HLS is

known in the chiral limit to have the vector manifestation fixed point to which hadronic matter is driven when chiral restoration takes place. We have argued that at this point,  $\sim 32$  light degrees of freedom become massless approaching  $T_c$  from below and account for the entropy found in lattice calculations. We have presented several cases where this scenario came out to be consistent with the notion of the VM, in particular in the form of “hadronic freedom” from the critical point down to what we called the “flash point.”

This prediction hangs crucially on the vector manifestation which holds strictly in HLS with the  $\rho$  and  $\pi$  and possibly the  $a_1$ . We should stress that this is not just an artifact of a theory. It should be falsifiable by lattice measurements of the vector meson mass near the critical point.

The principal conclusion that we arrive at is that physics is *continuous* across the chiral transition point  $T_c$  and  $n_c$ . We have given the argument that we see in RHIC data the indication that  $\sim 32$  light degrees of freedom go top-down across  $T_c$ , changing “smoothly” over from Wigner-Weyl mode to Nambu-Goldstone mode. In the modern parlance, we may call this “quark-hadron continuity” [104], but it has also been referred to as the “Cheshire cat principle” [87,105]. The earliest hint of this phenomenon was in the Bég-Shei theorem [5] mentioned above. How these modes wind up at  $T \gg T_c$  or  $n \gg n_c$  with a “perfect liquid” or a “color-flavor-locking” is a matter we have not addressed in this paper.

This “smooth” movement in the character of chiral symmetry is clearly manifest near nuclear matter density in that what represents Brown-Rho scaling is embedded in the Landau quasiparticle interactions. Even in such relativistic heavy-ion processes as CERES and NA60, there is no simple delineation of effects that signal how chiral symmetry manifests itself from mundane nuclear many-body effects.

There are several points we need to address to strengthen our main thesis.

The first is whether the VM argument made at one-loop order in the renormalization group equations survives at higher-loop order. Since there are no higher-loop calculations – a formidable task – at present, we cannot give a direct answer. However, we can make a convincing argument why the vector manifestation (VM) fixed point  $(g, a) = (0, 1)$  is unaffected by higher order graphs. One powerful way to show this is to use the protection by “enhanced symmetry” at the fixed point, but there is a simpler way to explain it. Although two-loop or higher-loop calculations are not available, it has been proven by Harada, Kugo and Yamawaki [106] that the tree-order low-energy theorems remain rigorously valid to all orders. In particular, the dimension-2 operators in the effective action remain the same to all orders. This means that the crucial relation in HLS, i.e.,  $m_\rho^2 = af_\pi^2 g^2$ , holds to all orders. The mass, therefore,

goes to zero as  $g$  goes to zero. Now the matching condition at the matching scale  $\Lambda$  says that  $g = 0$  when  $\langle \bar{q}q \rangle = 0$  and since  $g = 0$  is a fixed point of the RGE for  $g$  at any order (higher loops bring in higher powers in  $g$  in the beta function), it will flow to zero at the point where the condensate vanishes. One can also see that  $a = 1 + O(g^{2n})$  near  $\langle \bar{q}q \rangle = 0$  where  $n$  is the number of loops, so near  $T_c$  the correction to 1 is small and at  $T_c$ ,  $a = 1$ . Therefore we have the VM fixed point intact. We thus conclude that all our arguments made in the vicinity of  $T_c$  where the hadronic freedom is operative hold to all orders.

The next question is whether one cannot arrive at the VM in a QCD sum rule approach. In fact, the QCD sum rule calculation in medium by Hatsuda and Lee [107] is often cited as early theoretical evidence for a dropping vector meson mass in hot/dense matter, and since then there have been a large number of papers written on the subject, the most recent being in connection with the NA60 dimuon data [108]. As far as we know, there is only one publication on the subject in which the vector meson mass going to zero is *directly* associated with chiral restoration at  $T_c$  (in the chiral limit) [109]. Here the vanishing of the mass is attributed to the vanishing of the quark condensate, but it is not clear that that is associated with the vanishing of the vector coupling which is the origin of the VM.

Broadly speaking, we do expect, based on the recent development in holographic gravity-gauge duality, that one should in principle be able to arrive at the VM via QCD sum rules. In HLS/VM, the VM is established by equating the vector-vector correlator to the axia-vector-axial-vector correlator – matched between HLS and QCD at the matching scale  $\Lambda$  – when the quark condensate vanishes and then decimating the correlators à la Wilson to the relevant scale. What enables the Harada-Yamawaki theory to do this is the hidden local gauge invariance. Now it is plausible that hidden local symmetry results from *emerging* holography, in which case the problem could be addressed in terms of QCD sum rules exploiting the infinite tower of gauge fields for the correlators as suggested by Earlich et al. [110]. This also suggests that the holographic dual approach could unravel the structure of the states just below and just above  $T_c$  that we have discussed in terms of HLS/VM (below) and lattice indications (above).

## Acknowledgments

We are very grateful to the Bielefeld lattice gauge group of Frithjof Karsch, Dave Miller, Peter Petreczky, Olaf Kaczmarek and Felix Zantow, not only for the generous sharing of their  $SU(2) \times SU(2)$  full QCD, but also for helpful criticism and help and suggestions as to how to parameterize and how to understand their results. We are equally grateful to Masayasu Harada and

Chihiro Sasaki for their tuition on hidden local symmetry theory on which the developments described in this review are based. GEB was supported in part by the US Department of Energy under Grant No. DE-FG02-88ER40388. CHL was supported by the Korea Research Foundation Grant funded by the Korean Government(MOEHRD, Basic Research Promotion Fund) (KRF-2005-070-C00034).

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